Teachers' Edition

Investigating School Mathematics

Extending the Ideas

Investigating School Ma

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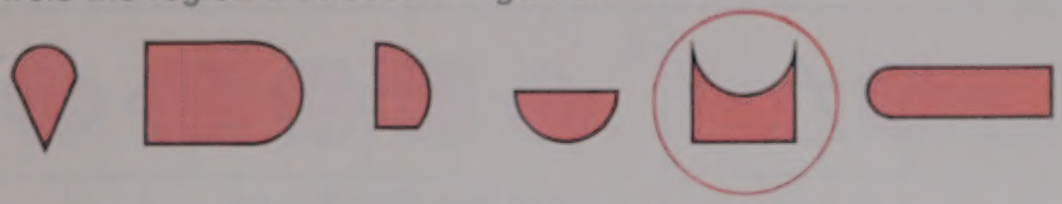
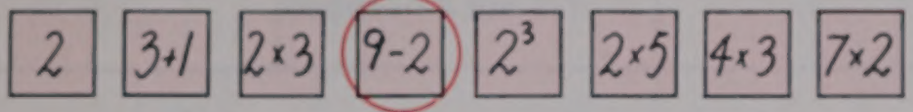
ABCDEFGHIJKL—WC—787654

Finding Differences

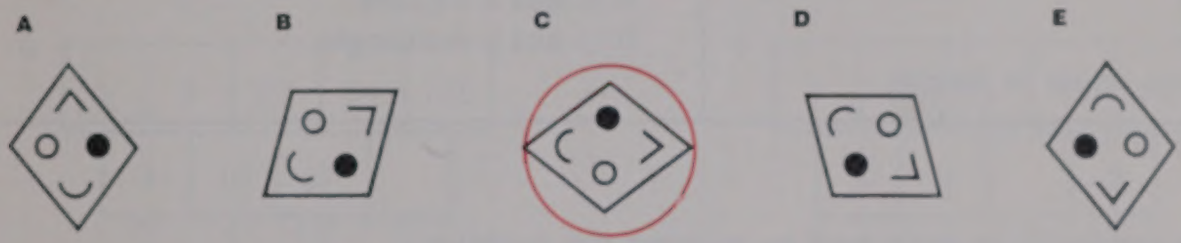
1. Circle the region that does not go with the others.



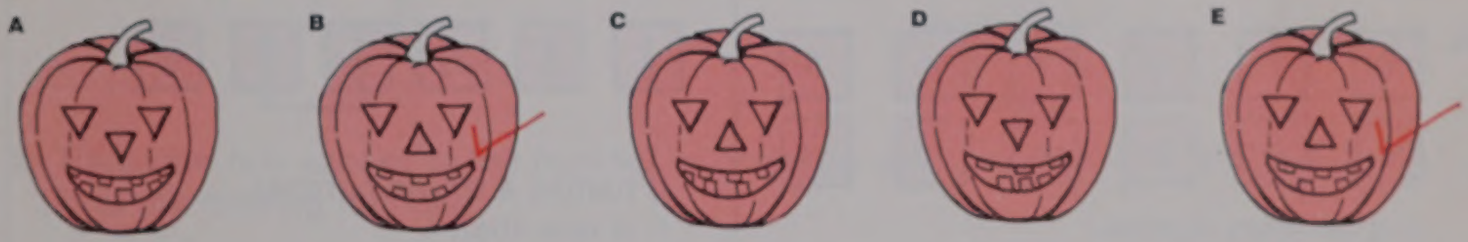
2. Which card does not belong in this set? Ring it.



3. Which figure is not identical to the others? Ring it.



4. Which two Jack O'Lanterns are the same? Check them (✓).



5. Which number in each set does not belong?

A {14, 8, 24, 0, 19, 32, 66, 48, 76} 19

B $\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{16}, \frac{7}{21}, \frac{8}{24}\right\}$ $\frac{6}{16}$

C {123, 132, 211, 213, 231, 312, 321} 211

D {30, 54, 90, 18, 27, 33, 6, 46, 24} 46

E $\left\{\frac{5}{3}, \frac{6}{2}, \frac{3}{4}, \frac{11}{5}, \frac{6}{5}, \frac{9}{4}, \frac{11}{7}, \frac{13}{10}\right\}$ $\frac{3}{4}$

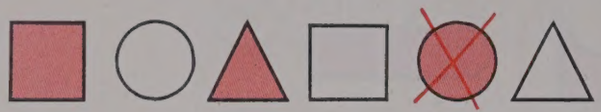
F $\left\{\frac{4}{2}, \frac{9}{3}, \frac{16}{4}, \frac{25}{5}, \frac{36}{6}, \frac{48}{7}, \frac{64}{8}, \frac{81}{9}\right\}$ $\frac{48}{7}$

Some children might be encouraged to make up their own problems similar to these.

Logical Reasoning

Mark an (X) on the object described.

1.

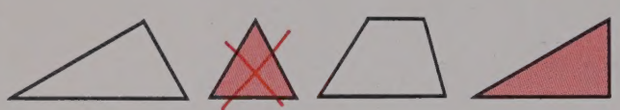


It is shaded.
It is **not** a square.
It is **not** a triangle.

2.

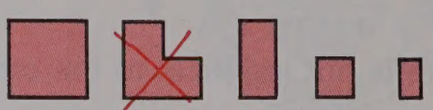
It has less than 8 sides.
It has more than 5 sides.
It is **not** a hexagon.

3.



It has 3 sides.
It is shaded.
All its sides are equal in length.

4.



It is **not** a square.
It is **not** a rectangle.

Complete the sentence in each part by naming the number.

5.

0	1	2	3	4	5
6	7	8	9	10	11

It is an even number.
It is greater than 6.
It is **not** divisible by 4.

It is 10.

6.

$\frac{7}{8}$	$\frac{8}{2}$	$2\frac{1}{3}$	$\frac{1}{5}$	$\frac{15}{5}$	$\frac{17}{4}$

It names a whole number.
It is less than 4.

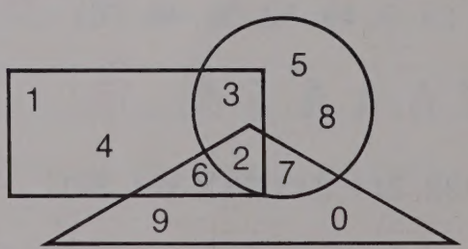
It is $\frac{15}{5}$.

7.

It is a 2-digit number.
The sum of the two digits is 13.
It is a square number.

It is 49.

8.



It is in the triangle.
It is **not** in the circle.
It is in the rectangle.

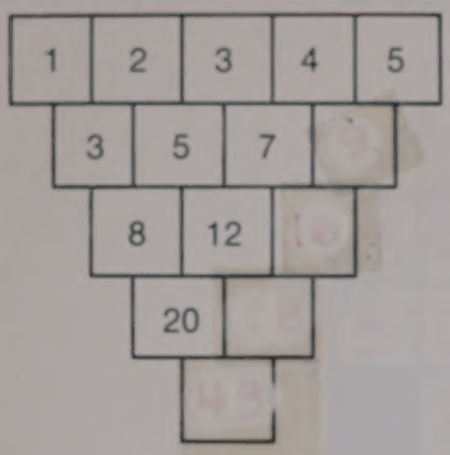
It is 6.

Suggest that children make a set of attribute cards using the figure of exercise 8 that will identify other numbers in the figure.

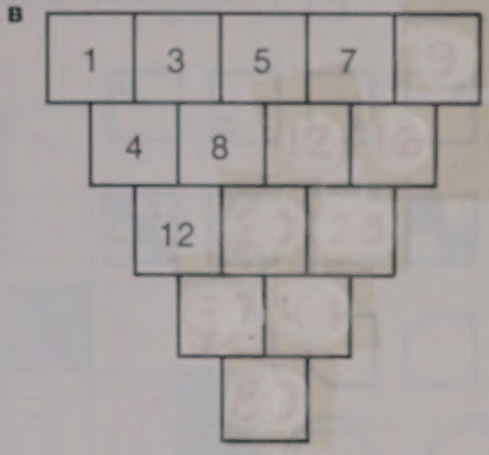
●Number Patterns

1. Find the pattern. Give the missing numbers in each square.

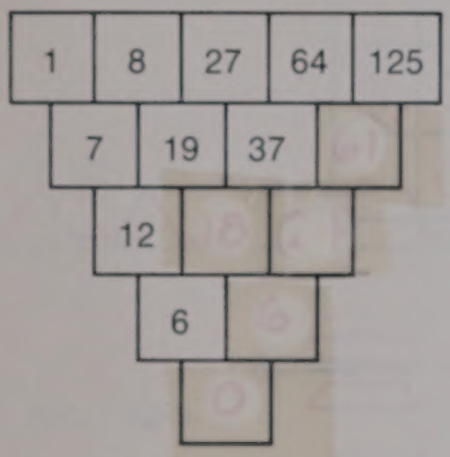
A



B



C



D

2. The same rule applies to the number in each row as the numbers in each column. Find the missing number.

A

5	3	8
7	4	11
12	7	

B

2	5	10
6	4	24
12	20	

C

	4	10
3	8	11
9	12	21

D

12	3	9
5	2	3
	1	6

E

720	48	15
30		5
24	8	3

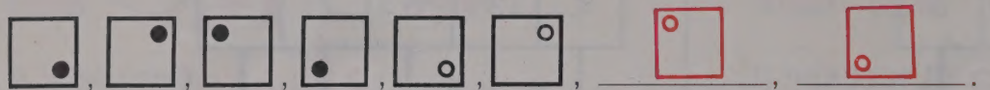
F

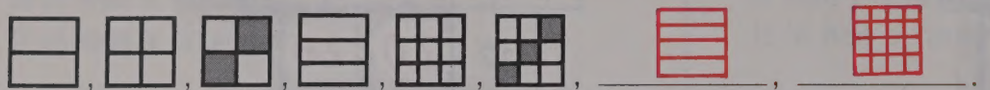
	2	10
4	3	12
20	6	120

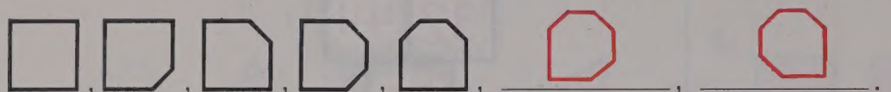
Figure Patterns

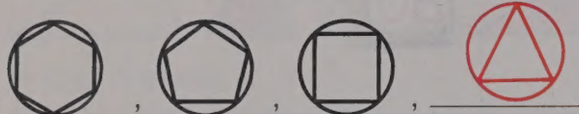
Continue the pattern for each blank.

1. E, m, ∃, ш, E, m, ∃.

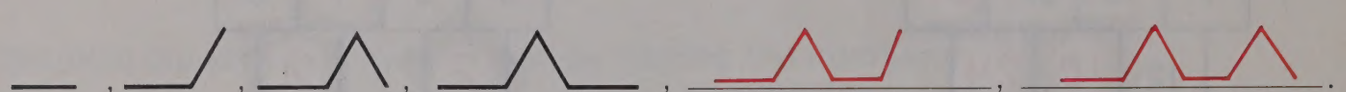
2. 

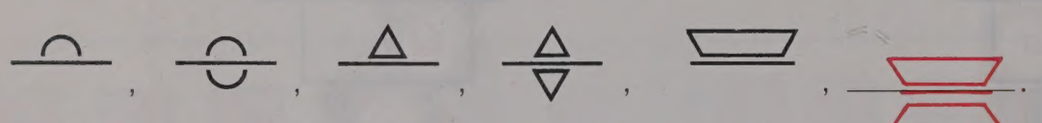
3. 

4. 

5. 

6. 

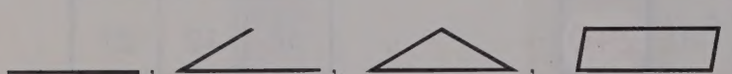
7. 

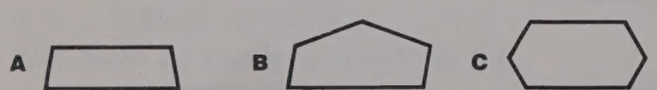
8. 

9. J, ℩, K, ⋊, L, ⅃, M, M.

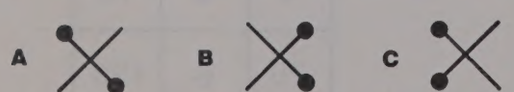
10. A, ∀, F, Ⅎ, P, ᑲ, T, ⊥.

Which figure **A**, **B** or **C** continues the pattern?

11.  ?B.

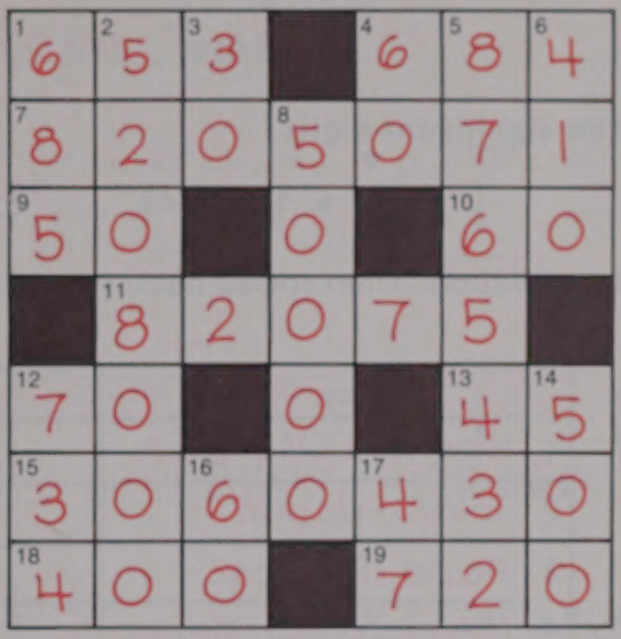


12.  ?C



Children will benefit from a discussion of the various patterns found in the problems on the page.

A CROSS-NUMBER PUZZLE



Across

1. 600 + 50 + 3
4. (6 × 100) + (8 × 10) + 4
7. Eight million, two hundred five thousand, seventy-one.
9. A 5 in the tens place means ___?___
10. Six tens
11. Eighty-two thousand, seventy-five
12. This number plus 3 down equals 10 × 10 .
13. Halfway between 4 tens and 5 tens.
15. Three million, sixty thousand, four hundred thirty
18. 449 rounded to the nearest hundred.
19. (7 × 100) + (2 × 10)

Down

1. The next number after 4 Across
2. Five million, two hundred seven thousand, nine hundred sixty-four rounded to the nearest thousand.
3. 3 × 10
4. Double of 3 Down
5. Eight million, seven hundred sixty-five thousand, four hundred thirty-two
6. The millions period of 927,410,806,743
8. 48,657 rounded to the nearest ten thousand.
12. (7 × 100) + (3 × 10) + 4
14. The Roman numeral for it is D
16. The number of seconds in a minute
17. Four tens and seven

Children who have not had experience with cross-number puzzles may need some instruction. Each blank square in the puzzle should be filled with 1 digit of the numeral answer.

●Rounding Large Numbers

We often round numbers to a certain number of **significant digits**.

242,746 rounded to **two** significant digits is 240,000.
5,849,213 rounded to **three** significant digits is 5,850,000.

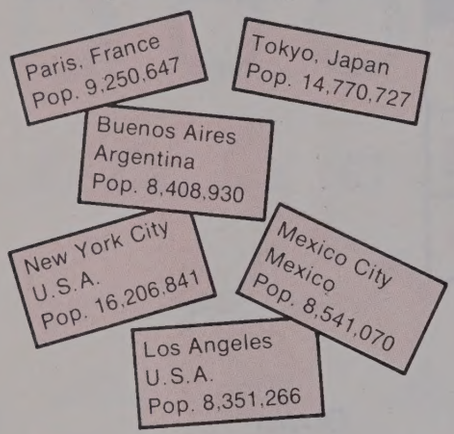
1. Round each number to **two** significant digits.

   A 742,966 740,000 B 58,929 59,000

2. Round each number to **three** significant digits.

   A 972,471 972,000 B 1,258,746 1,260,000

3. Rank the cities in order of population. Then round each number to **two** significant digits.



City	Rounded Population
1. New York City U.S.A.	16,000,000
2. Tokyo, Japan	15,000,000
3. Paris, France	9,300,000
4. Mexico City, Mexico	8,500,000
5. Buenos Aires, Argentina	8,400,000
6. Los Angeles, U.S.A.	8,400,000

4. Rank the Canadian provinces by their size from the largest to the smallest. Then round the area of each province to **two** significant digits.

Province	Area (km^2)	Rank	Rounded Area
Newfoundland	404,519	9	400,000
Prince Edward Island	5,655	12	5,700
Nova Scotia	55,490	11	55,000
New Brunswick	73,436	10	73,000
Quebec	1,540,687	2	1,500,000
Ontario	1,068,587	3	1,100,000
Manitoba	650,090	7	650,000
Saskatchewan	651,903	6	650,000
Alberta	661,188	5	660,000
British Columbia	949,600	4	950,000
Yukon Territory	536,326	8	540,000
Northwest Territories	3,379,698	1	3,400,000

The term significant digits will be new to most children. You may want to ascertain that children have completed exercise 1 and 2 correctly before completing the page.

●Powers of Ten

1. Study the example. Then complete each part in the same way.

EXAMPLE:

$10^2 \times 10^3 = 10^5$

$100 \times 1000 = 100{,}000$

A $10^2 \times 10^2 = 10^4$

100 × 100 = 10,000

B $10^3 \times 10^1 = 10^4$

1000 × 10 = 10,000

C $10^3 \times 10^4 = 10^7$

1000 × 10,000 = 10,000,000

D $10^5 \times 10^3 = 10^8$

100,000 × 1000 = 100,000,000

2. Solve each equation.

A $6 \times 10^2 = n$, $n =$ 600

B $n \times 10^3 = 7000$, $n =$ 7

C $4 \times n = 400$, $n =$ 10^2

D $7 \times 10^6 = n$, $n =$ 7,000,000

E $9 \times n = 900{,}000$, $n =$ 10^5

F $n \times 10^8 = 10^{10}$, $n =$ 10^2

3. Write each numerical fact in scientific notation.

A Light travels more than 200,000 kilometers in one second. 2×10^5 km/sec

B The moon is nearly 400,000 kilometers from earth. 4×10^5 km

C The planet Pluto is about 6,000,000,000 kilometers from the sun. 6×10^9 km

D The distance around the earth's equator is about 40,000 kilometers. 4×10^4 km

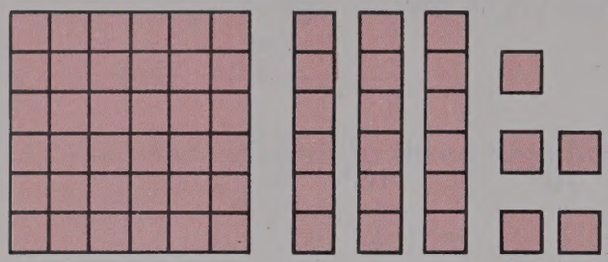
E This number is sometimes called a **Googol**: 1×10^{100}

1 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0
0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0
,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0
0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0
0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0

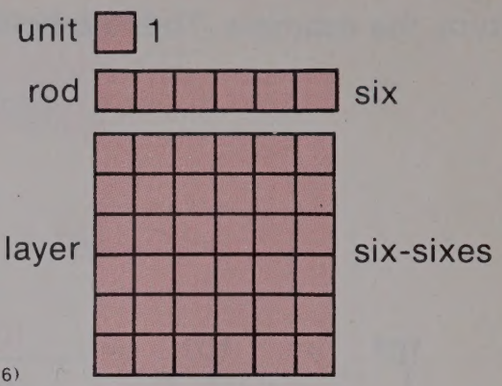
Base-Six Numerals

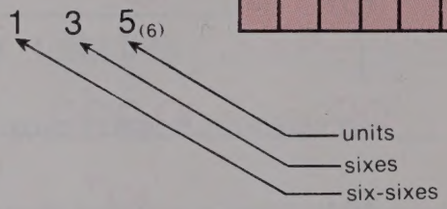
You can think about **units**, **rods**, and **layers** to help write base-six numerals.

EXAMPLE:



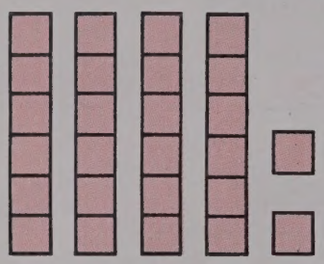
1 layer 3 rods 5 units





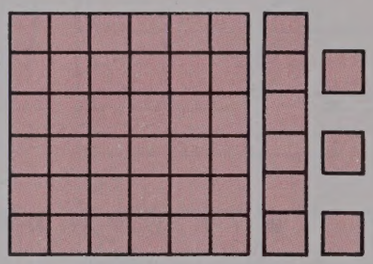
1. Give the base-six numeral for each part.

A



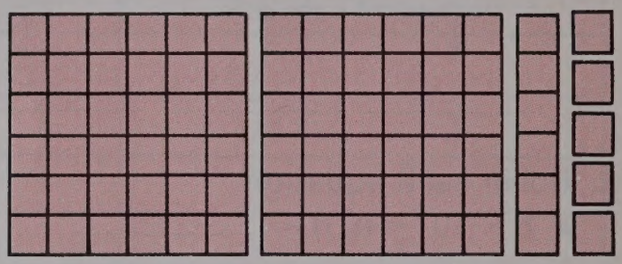
$42_{(6)}$

B



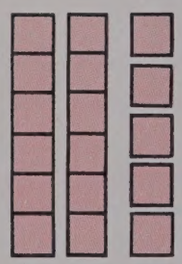
$113_{(6)}$

C



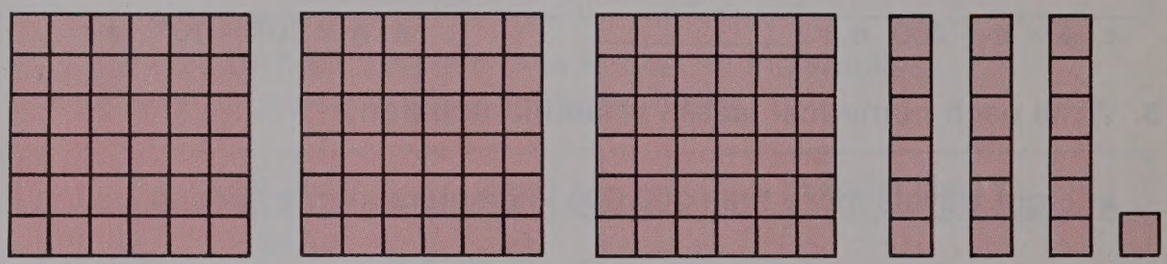
$215_{(6)}$

D



$25_{(6)}$

E



$331_{(6)}$

2. Write a base-six numeral for the total number shown in these parts.

A A and B $155_{(6)}$ B B and D $142_{(6)}$ C B and C $332_{(6)}$

D A and E $413_{(6)}$ E A and D $111_{(6)}$ F C and E $550_{(6)}$

3. Write a base-ten numeral for the number of squares shown in each part of Exercise 1.

A 26 B 45 C 83 D 17 E 127

4. Write a base-ten numeral for each answer of Exercise 2.

A 71 B 62 C 128 D 153 E 43 F 210

Units, rods, and layers cut from graph paper can be used to illustrate base-six numerals. Multibase blocks, if available, could be used to an advantage with this page.

Each problem requires two operations. Give the letters for the operations; A for Addition, S for Subtraction, M for Multiplication, and D for Division, in the *order* you would use them.

Some answers may vary.

1. Had ▒ dollars.
   Spent ▒ for a book.
   Spent ▒ for a pencil.
   How much money was left? A,S

2. Worked ▒ hours one day.
   Worked ▒ hours another day.
   Earned ▒ dollars an hour.
   How much earnings in all? A,M

3. Had ▒ jellybeans. Ate ▒.
   Gave the rest to ▒ friends.
   Each got the same number.
   How many did each friend get? S,D

4. Carol: ▒ centimeters tall.
   Jan: ▒ centimeters tall.
   Carol grew ▒ centimeters.
   Jan grew ▒ centimeters.
   How much taller was the taller girl? A,S

5. Bought ▒ writing tablets at ▒ cents each.
   Sales tax: ▒ cents.
   How much in all? M,A

6. Drove ▒ kilometers at ▒ kilometers per hour.
   Stopped ▒ hours for lunch.
   How much time in all? D,A

7. Complete each table. Avoid as much computation as possible.

A

+	76	569	0	129
129	205	698	129	258
76	152	645	76	205
569	645	1138	569	698
0	76	569	0	129

B

×	23	64	0	49
49	1127	3136	0	2401
23	529	1472	0	1127
1	23	64	0	49
64	1472	4096	0	3136

The sums and products in exercise 7 can be found in the table by using the commutative principles or the 0 and 1 principles.

Coded Equations

Solve the equations. Find the number for your solution in the important message. Write the letter of the problem in the blank above it.

A $40 \div 4 = n$

$n =$ 10

B $n + 0 = 18$

$n =$ 18

C $73 \times n = 73$

$n =$ 1

D $2^2 \times 5 = n$

$n =$ 20

E $27 \div 9 = n$

$n =$ 3

F $n^2 = 81$

$n =$ 9

G $6 \times n^2 = 24$

$n =$ 2

H $32 \div n = 2$

$n =$ 16

I $7 \times n = 77$

$n =$ 11

J $n \times 10 = 50$

$n =$ 5

K $(6 \times 4) - 5 = n$

$n =$ 19

L $25 - n = 12$

$n =$ 13

M $n^2 = 49$

$n =$ 7

N $45 \div 3 = n$

$n =$ 15

O $56 \div 2 = n$

$n =$ 28

P $9 + n = 13$

$n =$ 4

Q $(30 \div 3) \times 3 = n$

$n =$ 30

R $(48 \div n) \times 8 = 48$

$n =$ 8

S $(88 - 24) + n = 88$

$n =$ 24

T $(n - 33) + 33 = 39$

$n =$ 39

U $10 = n - 7$

$n =$ 17

V $100 \div n = 4$

$n =$ 25

W $7 \times n = 42$

$n =$ 6

X $42 \div n = 2$

$n =$ 21

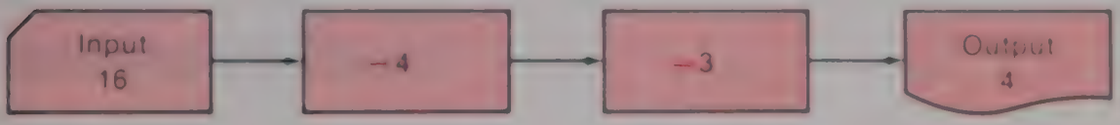
IMPORTANT MESSAGE

D	O	N	'	T		G	R	O	W			
20	28	15		39		2	8	28	6			
O	L	D	-	S		M	O	K	E			
28	13	20		24		7	28	19	3			
C	I	G	A	R	E	T	T	E	S	!		
1	11	2	10	8	3	39	39	3	24			

Flow Chart Operations

Study the example. Then give the missing operation signs. $-$, $-$, $-$, $-$ for each exercise.

EXAMPLE

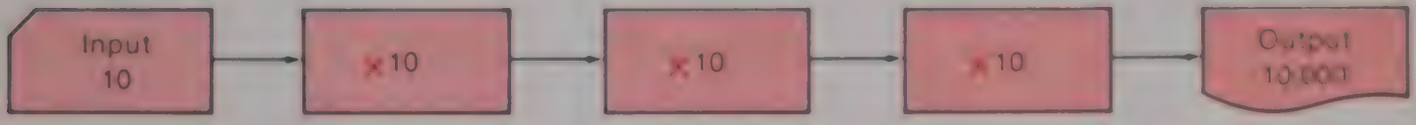
1. 

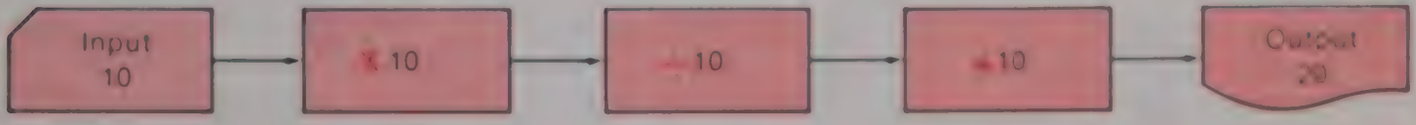
2. 

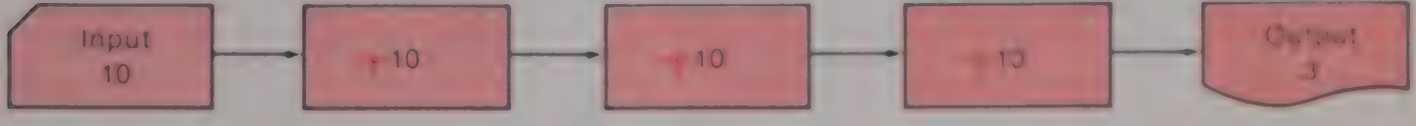
3. 

4. 

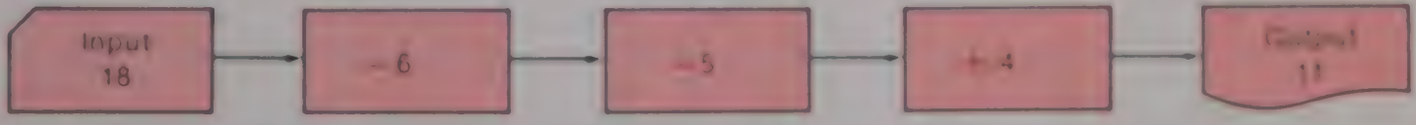
5. 

6. 

7. 

8. 

9. 

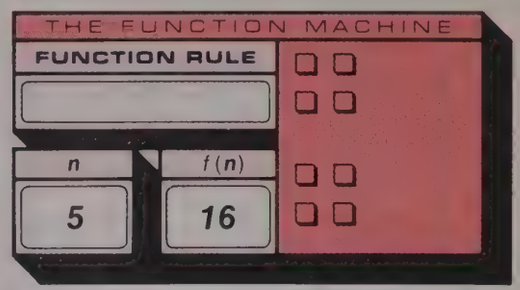
10. 

Children who are unfamiliar with flow charts may need some help in completing this page.

Guess the Function Rule

Study each function table and find the function rule. More than one operation is used in some tables.
The function rule for the numbers in the table is:
$f(n) = (3 \times n) + 1$

Function Rule	
$(3 \times n) + 1$	
1	4
2	7
3	10
4	13
5	16



1.

Function Rule	
$(3 \times n) - 1$	
n	$f(n)$
1	2
2	5
3	8
4	11
5	14

2.

Function Rule	
$(10 \times n) - 1$	
n	$f(n)$
2	21
5	51
7	71
10	101
23	231

3.

Function Rule	
$(n \div 3) + 1$	
n	$f(n)$
3	2
12	5
15	6
24	9
33	12

4.

Function Rule	
$n \times n$	
n	$f(n)$
1	1
4	16
7	49
9	81
10	100

5.

Function Rule	
$(n \times n) + 2$	
n	$f(n)$
1	3
4	18
5	27
8	66
10	102

6.

Function Rule	
$n \times n \times n$	
n	$f(n)$
1	1
2	8
3	27
4	64
5	125

Special Products and Factors

Complete each flow chart.

1. Input 20 → · 30 → Output 600

2. Input 800 → ÷ 4 → Output 200

3. Input 200 → · 20 → Output 4000

4. Input 4200 → ÷ 60 → Output 70

5. Input 90 → · 200 → Output 18.000

6. Input 16.000 → ÷ 80 → Output 200

7. Input 540 → ÷ 60 → Output 9

8. Input 10^2 → × 10^3 → Output 10^5

9. Input 700 → · 60 → Output 42.000

10. Input 18 → · 300 → Output 5400

11. Complete each table.

A.

Products		Quotients
800	40 ÷ 20	2
1200	60 ÷ 20	3
1000	100 ÷ 10	10
400	100 ÷ 4	25
32.000	400 ÷ 80	5
160.000	4000 ÷ 40	100
18.000	900 ÷ 20	45

B.

Products		Quotients
2400	120 ÷ 20	6
100.000	2000 ÷ 50	40
70.000	700 ÷ 100	7
120	60 ÷ 2	30
4800	240 ÷ 20	12
1.000.000	1000 ÷ 1000	1
640	80 ÷ 8	10

12. If you multiply 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 the product will be either

A 362.880 B 3.628.800 C 36.288.000 D 362.880.000

Can you find which product is correct without actually multiplying? B

Note in exercise 12 that the factors 2, 5, and 10 have a product of 100, while the remaining factors will not give a multiple of 10. Therefore the product must end in two zeros.

Problems, Flow Charts and Equations

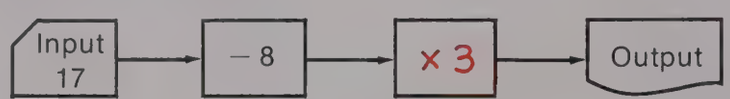
Complete each flow chart and solve an equation for the problem.

1. Problem: If 7 is multiplied by 9 and 5 subtracted from the product, what is the resulting number?

Flow Chart: 

Equation: $(7 \times 9) - 5 = n$ $n =$ 58

2. Problem: If 8 is subtracted from 17 and the difference multiplied by 3, what is the result?

Flow Chart: 

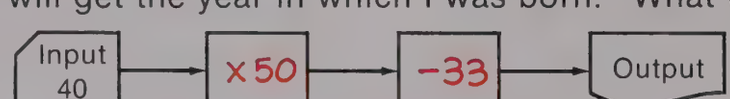
Equation: $(17 - 8) \times 3 = n$, $n =$ 27

3. Problem: If 13 is added to 23 and the sum is divided by 9, what is the result?

Flow Chart: 

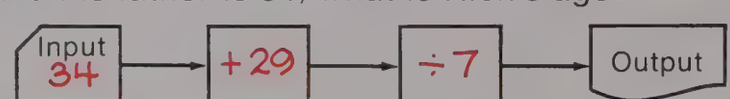
Equation: $(23+13) \div 9 = n$ $n = 4$

4. Problem: Andrea said "If you multiply 40 by 50 and then subtract 33 you will get the year in which I was born." What was that year?

Flow Chart: 

Equation: $(40 \times 50) - 3 = n$ $n = 1967$

5. Problem: Rick said "If you add my mother's age to my father's age and divide by 7 you will get my age." If Rick's mother is 29 and his father is 34, what is Rick's age?

Flow Chart: 

Equation: $(34+29) \div 7 = n$ $n = 9$

The problems are arranged so that the student must do a greater amount in each successive problem.

Interesting Intersections

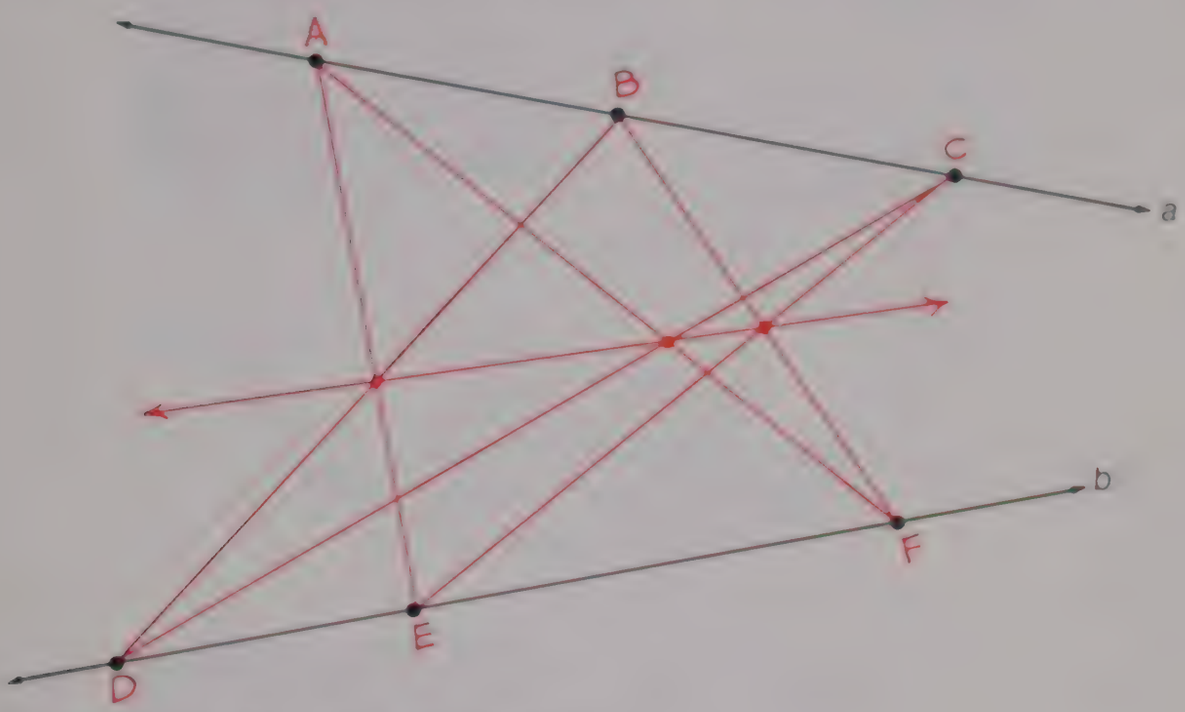
1. Mark points *A*, *B*, and *C* on line *a* in the order named.

2. Mark points *D*, *E*, and *F* on line *b* in the order named.

3. Draw these segments: *AE*, *AF*, *BD*, *BF*, *CD*, *CE*.

4. Mark the intersection of:
   - A *AE* and *BD*
   - B *AF* and *CD*
   - C *BF* and *CE*

   (These three points should all lie on a line. Draw this line.)



5. Draw two more lines. Try this activity again.

This activity is based upon a famous geometric theorem of Pascal (1623–1662).

Figuring the Angles

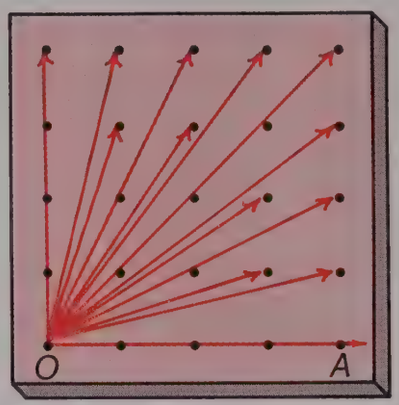
1. $\overrightarrow{OA}$ is one side of an angle on the geoboard.

   A Show as many other rays from O as you can. Each ray must have at least one other point of the geoboard in it other than point O.

   B How many angles with $\overrightarrow{OA}$ as one side have you drawn? 12

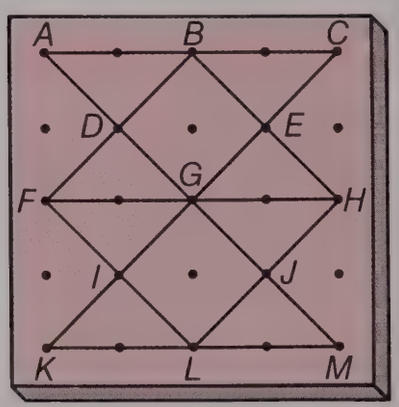
   C How many different angles can you count using the rays you drew on the geoboard? 78

   answers will vary

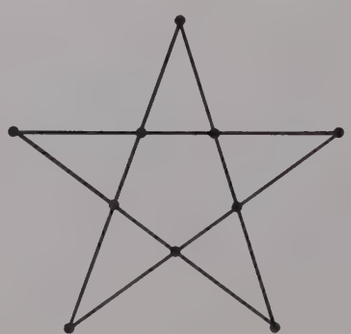


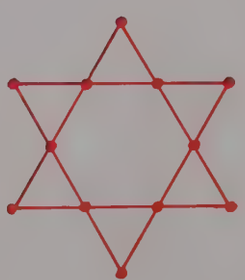
2. Using the lettered points and the segments shown on the geoboard, find and name as many triangles as you can.

   ABD, BCE, ACG, FGD, GHE,
   FHB, FGI, GHJ, FHL, KLI,
   LMJ, KMG

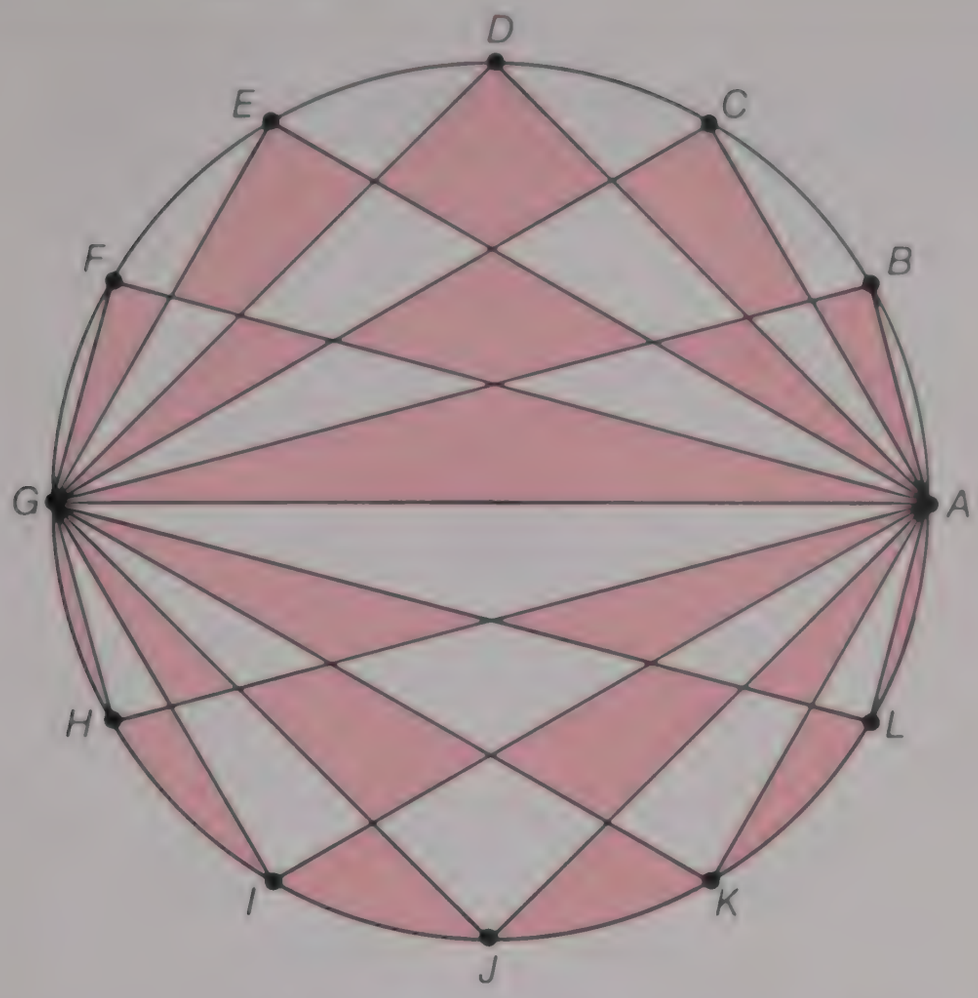


3. The figure at the right shows how 10 points can be placed in 5 rows with 4 points on each row.
   Can you show how 12 points can be placed in 6 rows with 4 points in each row?





A Search for Special Figures



1. Parallel segments lie in parallel lines. List as many pairs of parallel segments as you can find in the drawing above.

$GH \parallel BA$	$\overline{FG} \parallel \overline{AL}$	$\overline{EG} \parallel \overline{AK}$	$\overline{DG} \parallel \overline{AJ}$	$\overline{CG} \parallel \overline{AI}$
$\overline{BG} \parallel \overline{AH}$	$\overline{LG} \parallel \overline{AF}$	$\overline{KG} \parallel \overline{AE}$	$\overline{JG} \parallel \overline{AD}$	$\overline{IG} \parallel \overline{AC}$

2. Perpendicular segments intersect to form right angles. List as many pairs of perpendicular segments as you can find in the drawing. Use a small file card to help you find the perpendicular segments.

$GF \perp FA$	$\overline{GE} \perp \overline{EA}$	$\overline{GD} \perp \overline{DA}$	$\overline{GC} \perp \overline{CA}$	$\overline{GB} \perp \overline{BA}$
$\overline{GH} \perp \overline{HA}$	$\overline{GI} \perp \overline{IA}$	$\overline{GJ} \perp \overline{JA}$	$\overline{GK} \perp \overline{KA}$	$\overline{GL} \perp \overline{LA}$

3. Can you find a square in the drawing? Give its vertices. GDAJ

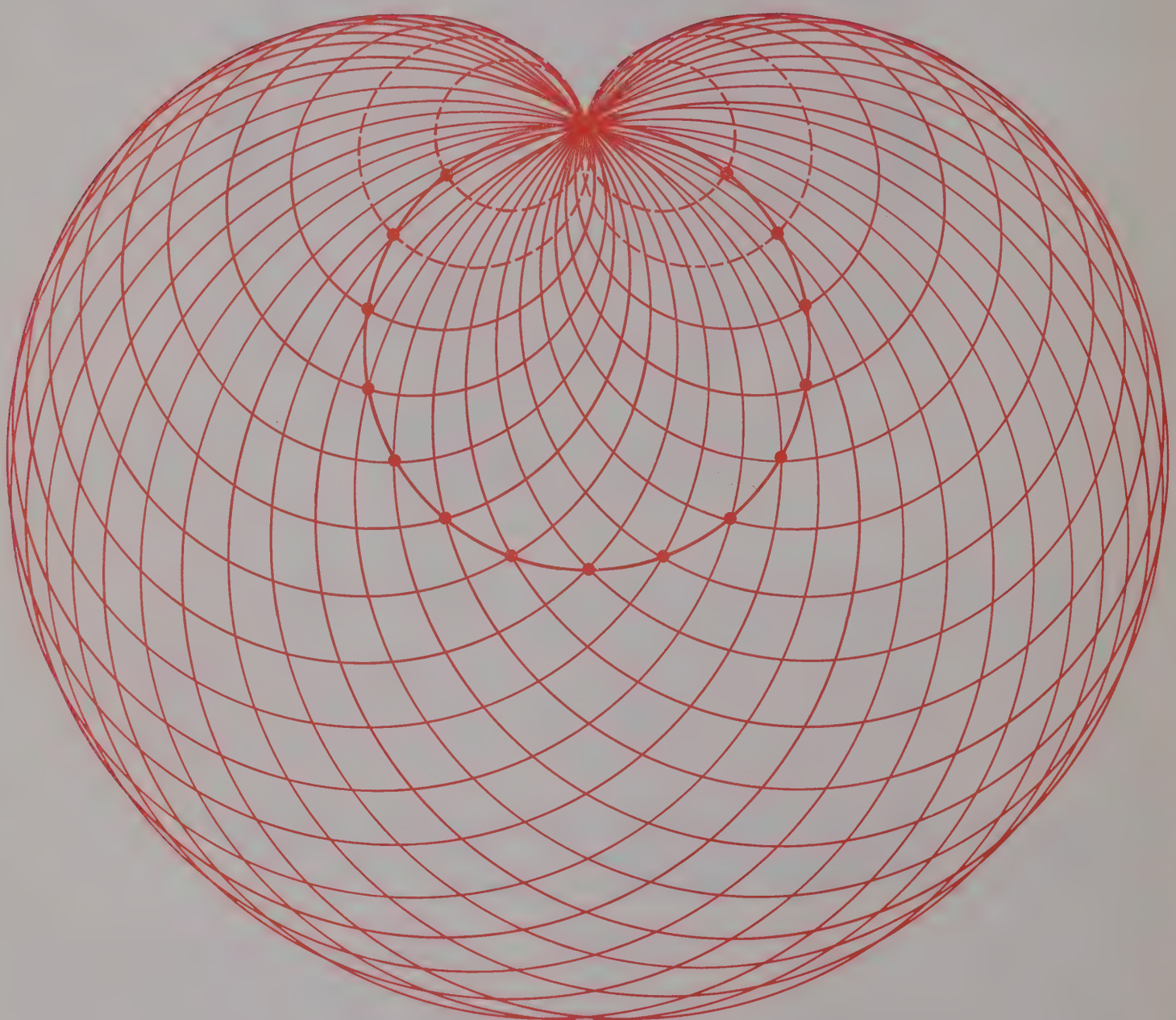
4. How many rectangles can you find in the drawing? Name them.

GBAH	GFAL	GKAE	GDAJ	GCAI

It is not necessary that children find all pairs of parallel segments or all pairs of perpendicular segments.

Circles to Curves

1. Use a compass to draw circles whose centers are on the points marked on the solid red circle. Each circle must also pass through point X on the circle. The curve that all the circles touch is called a cardioid. The dashed red circles are shown to help get you started.

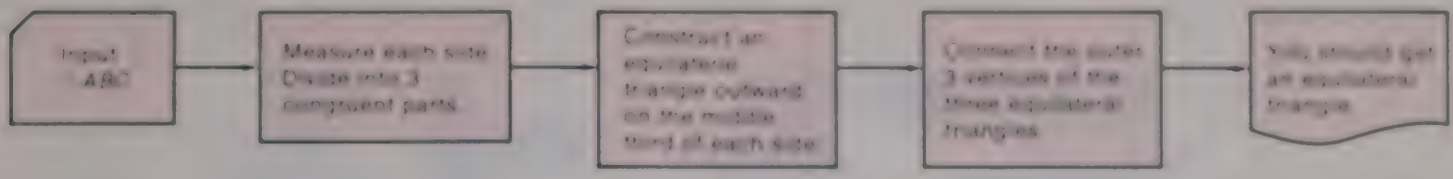


2. On another sheet of paper see if you can make a different design involving circles and curves.

You can suggest that children color or shade the cardioid design to highlight some of its features.

Measuring Segments and Angles

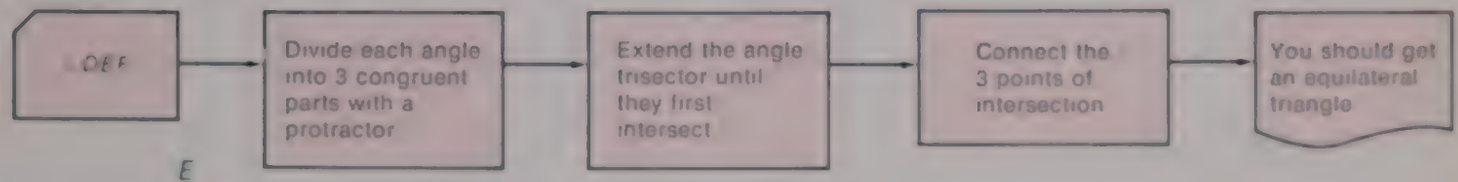
1. Follow the flow chart instructions.

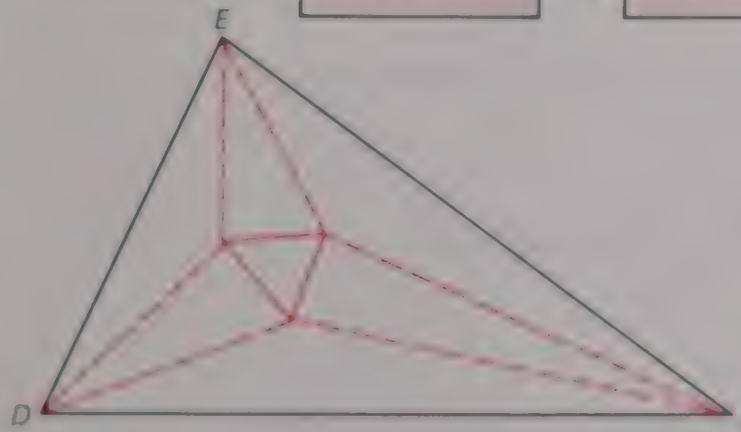


2. What is the perimeter of $\triangle ABC$? 24.9 cm

3. What is the perimeter of the equilateral triangle you constructed? 22.5 cm

4. Follow the flow chart instructions.





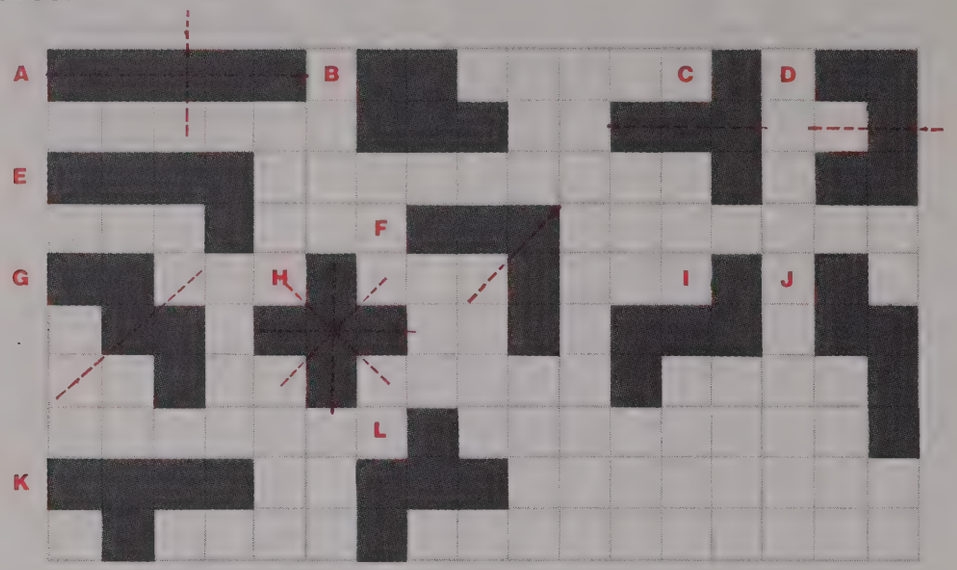
5. What is the perimeter of $\triangle DEF$? 21.2 cm

6. What is the perimeter of the equilateral triangle you found? 3.9 cm

Suggest to the children that the segments in the drawings be measured to the nearest tenth of a centimeter.

Pentomino Pieces

The twelve regions below are the pentomino regions. Each region is formed with 5 unit squares.



1. Give the number of lines of symmetry of each piece. Show the lines on the pieces above.

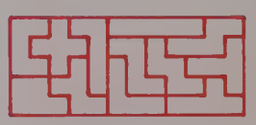
A	C	E	G	I	K
2	1	0	1	0	0

B	D	F	H	J	L
0	1	1	4	0	0

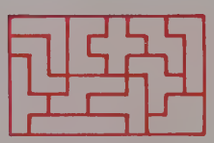
2. All of the pieces have the same area. Eleven of the twelve pieces have the same perimeter. What is this perimeter? 12

3. What one piece has a perimeter different from the others? What is its perimeter? 10

4. Can you make a 5 by 12 rectangle using all 12 pentomino pieces?



5. Try making a 6 by 10 rectangle using all 12 pentomino pieces.



6. Cut out the 12 pentomino pieces from graph paper. Try to arrange the pieces into a 3 by 20 unit rectangle.

Exercises 4, 5, and 6 are challenging problems. You may want to make permanent models of the pentomino pieces and put these puzzles in a "Math Puzzle" collection.

Tesselations

Each shaded region repeated several times will exactly cover the polygon beside it. Show, by shading or coloring the polygon, how this can be done.

Answers may vary on some tesselations.

1. 

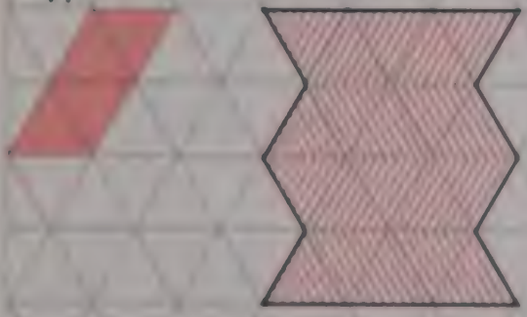
2. 

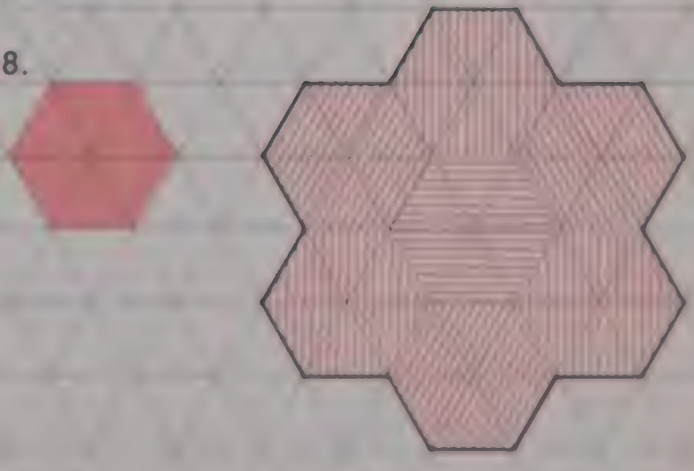
3. 

4. 

5. 

6. 

7. 

8. 

Some of the regions can be covered in more than one way by the given figure.

Palindromic Numerals

A **Palindrome** is a numeral that represents the same number when its digits are reversed. The numerals 181, 2112 and 72527 are examples of palindromes. Study the example to see how to get a palindrome by reversing digits and adding.

Starting Number →	438
Reverse →	+ 834
Sum →	1272
Reverse →	+ 2721
Palindrome →	3993

Find palindromes for each of the following numbers. Some will require several reversals.

1.
871
+178
1049
+9401
10450
+05401
15851

2.
2743
+3472
6215
+5126
11341
+14311
25652

3.
69
+96
165
+561
726
+627
1353
+3531
4884

4.
683
+386
1069
+9601
10670
+07601
18271
+17281
35552
+25553
61105
+50116
111221
+122111
233332

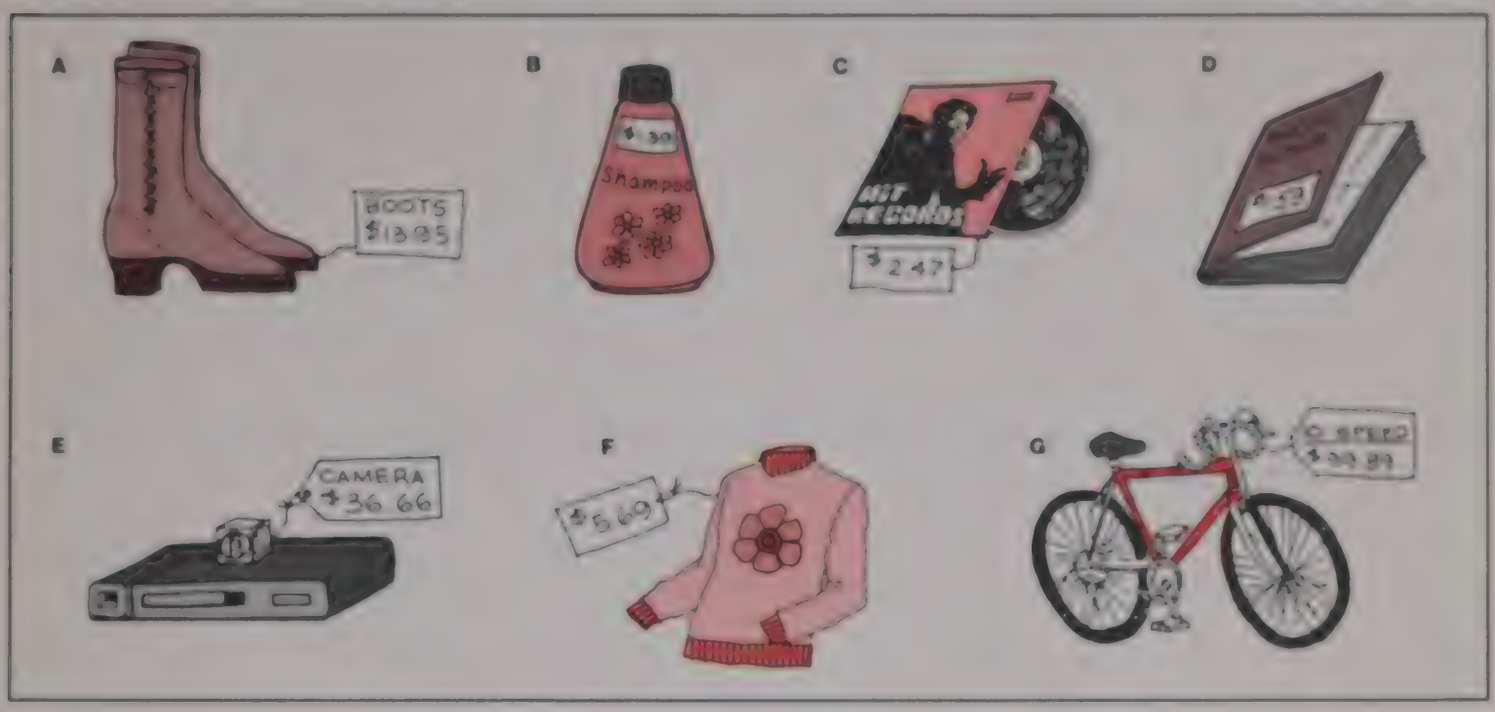
5.
987
+789
1776
+6771
8547
+7458
16005
+50061
66066

6.
6524
+4256
10780
+08701
19481
+18491
37972
+27973
65945
+54956
120901
+109021
229922

Try to make your own palindrome by this method. Warning: Do not start with 196.

It is not known if 196 will produce a palindrome. It has been carried out through thousands of reversals without producing a palindrome.

Money Problems



1. How much for B bottle of shampoo and a photo album? $2.98

2. Carlos bought a record. How much change should he get from a $5 bill? $2.53

3. Andrea bought the photo album. The sales tax was 8¢.
   - A What was the total amount? $1.67
   - B How much money should she get in change if she paid for the album with $2.00? $.33

4. Pam is saving her money to buy a bicycle. She has saved $43.77. How much more must she save to buy the one shown? $56.12

5. Mr. Walters bought the camera when it was on sale for $29.95. How much did he save? $6.71

6. Valerie bought the boots and the sweater. The sales tax was 98¢. She gave the salesperson a $20 bill. Was that enough? No
   How much or too little was $20.00? $.62 to little

7. A If you had $10 how many records could you buy? 4
   B How much money would you have left? $.12

8. What is the total value of all of articles shown above? $161.64

Multiplying

Find the two products in each part of exercises 1 through 4.

1. A
$$\begin{array}{r} 12 \\ \times\ 42 \\ \hline 504 \end{array}$$

B
$$\begin{array}{r} 21 \\ \times\ 24 \\ \hline 504 \end{array}$$

2. A
$$\begin{array}{r} 24 \\ \times\ 63 \\ \hline 1512 \end{array}$$

B
$$\begin{array}{r} 42 \\ \times\ 36 \\ \hline 1512 \end{array}$$

3. A
$$\begin{array}{r} 82 \\ \times\ 14 \\ \hline 1148 \end{array}$$

B
$$\begin{array}{r} 28 \\ \times\ 41 \\ \hline 1148 \end{array}$$

4. A
$$\begin{array}{r} 96 \\ \times\ 46 \\ \hline 4416 \end{array}$$

B
$$\begin{array}{r} 69 \\ \times\ 64 \\ \hline 4416 \end{array}$$

5. Can you find some more multiplication problems like the ones in Exercises 1–4? Show your examples. Answers will vary

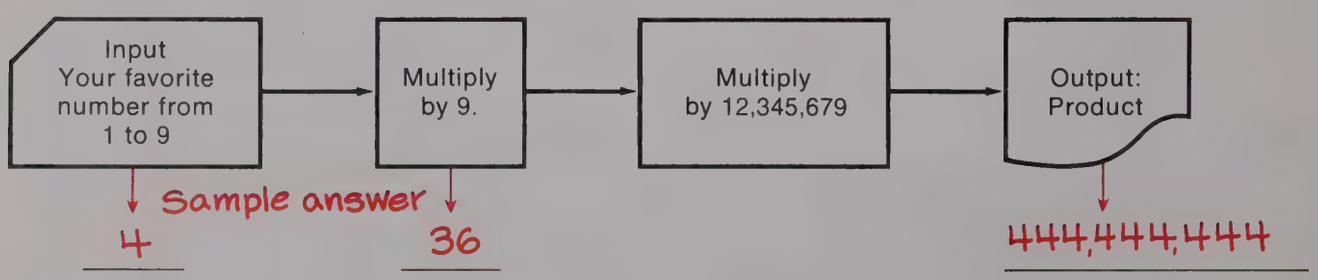
6. Find the missing digits in each of these multiplication problems.

A
$$\begin{array}{r} 99 \\ \times\ 23 \\ \hline 297 \\ 1980 \\ \hline 2277 \end{array}$$

B
$$\begin{array}{r} 359 \\ \times\ 77 \\ \hline 2513 \\ 25130 \\ \hline 27643 \end{array}$$

C
$$\begin{array}{r} 476 \\ \times\ 999 \\ \hline 4284 \\ 42840 \\ 428400 \\ \hline 475524 \end{array}$$

7. Try this flow chart problem.



What do you notice about the output number? 9 digits all the same as the input number

Try another input number.

Note that in exercise 5 the product of the units digits must equal the product of the tens digits in order for the reversals to give the same product.

Squares and Cubes

Use the table of squares and cubes to solve the equations.

Number n	Square n^2	Cube n^3	Number n	Square n^2	Cube n^3
1	1	1	26	676	17 576
2	4	8	27	729	19 683
3	9	27	28	784	21 952
4	16	64	29	841	24 389
5	25	125	30	900	27 000
6	36	216	31	961	29 791
7	49	343	32	1024	32 768
8	64	512	33	1089	35 937
9	81	729	34	1156	39 304
10	100	1000	35	1225	42 875
11	121	1331	36	1296	46 656
12	144	1728	37	1369	50 653
13	169	2197	38	1444	54 872
14	196	2744	39	1521	59 319
15	225	3375	40	1600	64 000
16	256	4096	41	1681	68 921
17	289	4913	42	1764	74 088
18	324	5832	43	1849	79 507
19	361	6859	44	1936	85 184
20	400	8000	45	2025	91 125
21	441	9261	46	2116	97 336
22	484	10 648	47	2209	103 823
23	529	12 167	48	2304	110 592
24	576	13 824	49	2401	117 649
25	625	15 625	50	2500	125 000

A $17^2 =$ 289

B $37^2 =$ 1369

C $42^2 =$ 1764

D $7^3 =$ 343

E $21^3 =$ 9261

F $46^2 =$ 2116

G $19^3 =$ 6859

H $24^2 =$ 576

I $47^3 =$ 103,823

J $35^2 =$ 1225

2. Find the number for n in each equation.

A $n^2 = 484$

$n =$ 22

B $n^2 = 676$

$n =$ 26

C $n^2 = 2401$

$n =$ 49

D $n^2 = 1024$

$n =$ 32

E $n^3 = 343$

$n =$ 7

F $n^3 = 1331$

$n =$ 11

G $n^3 = 13\,824$

$n =$ 24

H $n^3 = 50\,653$

$n =$ 37

I $n^3 = 103\,823$

$n =$ 47

3. Study the example. Then solve the equations by using the table.

EXAMPLE:

$n^2 = 6^2 + 8^2$

$n^2 = 36 + 64$

$n^2 = 100$

$n = 10$

A $n^2 = 3^2 + 4^2$ $n = 5$

B $n^2 = 12^2 + 5^2$ $n = 13$

C $n^2 = 20^2 + 21^2$ $n = 29$

D $n^2 = 9^2 + 40^2$ $n = 41$

E $n^2 = 10^2 + 24^2$ $n = 26$

4. Does the pattern continue? Write the next three equations.

$1^3 = 1^2$

$1^3 + 2^3 = 3^2$

$1^3 + 2^3 + 3^3 = 6^2$

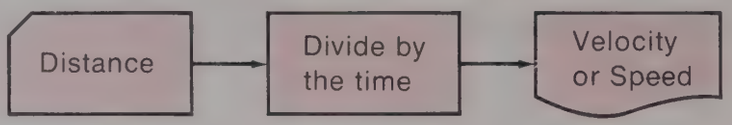
A $1^3+2^3+3^3+4^3 = 10^2$

B $1^3+2^3+3^3+4^3+5^3 = 15^2$

C $1^3+2^3+3^3+4^3+5^3+6^3 = 21^2$

You might discuss the use of the table of squares and cubes before trying the exercises.

Velocity Problems



1. A spine tailed swift could fly 510 kilometers in 3 hours. What is the speed of the swift? 170 km/h

2. If a sailfish could swim at top speed for 5 hours it could swim 540 kilometers. What would the velocity be? 180 km/h

3. A monarch butterfly could fly 256 kilometers in 8 hours. What speed is this? 32 km/h

4. If a pronghorn antelope could run 336 kilometers in 6 hours, what would the velocity be? 56 km/h

5. The fastest of all snakes is the black mambia. If a black mambia wriggled for 12 hours it would travel about 132 kilometers. What speed would this be? 11 km/h

6. If a giant tortoise traveled at top speed it could only travel 1918 meters in 7 hours. What is this speed in meters per hour? 274 m/h

7. A very fast spider might travel 2340 centimeters in 45 seconds. What is this speed in centimeters per second? 52 cm/s

8. If a honey bee traveled at "cruising speed" for 9 hours it might travel about 198 kilometers. What speed is "cruising speed?" 22 km/h

9. A very slow snail might take 12 hours to travel 696 centimeters. What is the speed of a slow snail? 58 cm/h

10. A fast snail might be able to travel 696 centimeters in 8 minutes. What is its speed? 87 cm/min

11. Find a speed about yourself such as walking, swimming, running or bicycling. answers will vary

Arithmetic Mean

1. Frank and Karen played miniature golf. Find their total score and the missing score.

Hole	1	2	3	4	5	6	7	8	9	Total	Average
Karen	3	2	4	1	4	4	3	4	2	27	3
Frank	5	4	6	4	3	2	3	6	3	36	4

2. Find the missing scores and averages.

A

Home	Opponents
14	13
21	34
20	17
33	12
Averages 22	19

B

Home	Opponents
10	13
27	12
24	21
19	14
Averages 20	15

C

Home	Opponents
34	7
19	21
16	20
27	0
Averages 24	12

3. Make up baseball and basketball scores so the averages given are correct. Answers will vary

Sample answers

Baseball

A

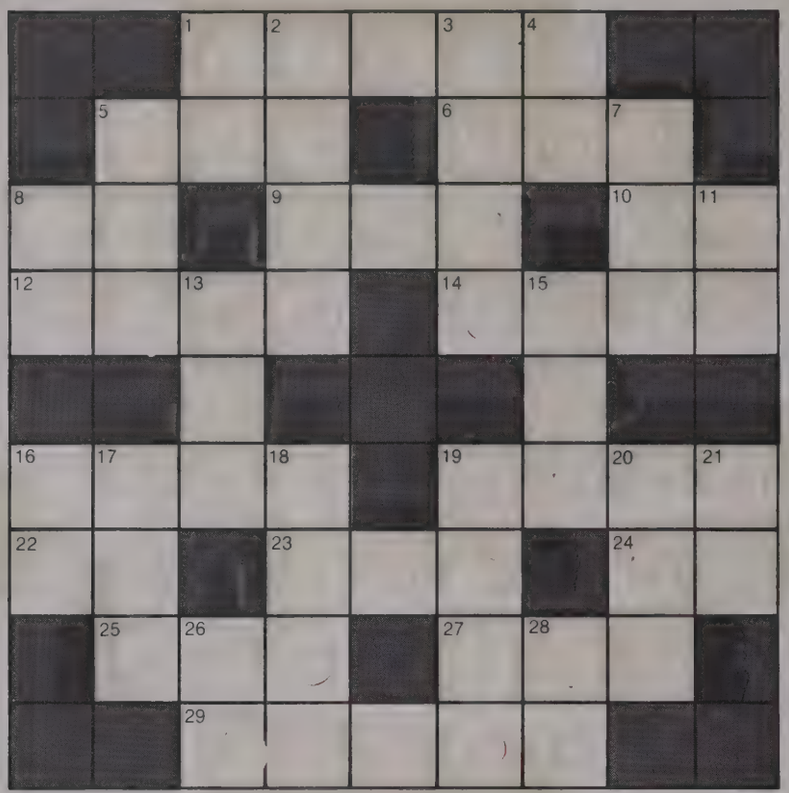
Home	Opponents
3	1
5	2
7	6
1	3
Averages 4	3

Basketball

B

Home	Opponents
52	66
36	34
75	27
25	29
Averages 47	39

A Division Cross-Number Puzzle



Across

1. Find the dividend: $92\overline{)123}$

5. $9\overline{)2196}$

6. $8109 \div 9$

8. The remainder for $376 \div 21$

9. $8694 \div 42$

10. $2^5 \div 2$

12. $n \div 4 = 1022$

14. What is the dividend? $14\overline{)451}$

16. $n \div 67 = 67$

19. Find the dividend: $74\overline{)47}$

22. $600 \div 20$

23. $46{,}332 \div 66$

24. The remainder for $2056 \div 72$

25. $(3 \times 13 \times 100) \div 10$

27. $n \div 9 = 13$

29. $10^6 \div 10^2$

Down

1. $56 \div 4$

2. $142{,}800 \div 100$

3. $49{,}400 \div 15$

4. $n \div 4 = 15$

5. $5800 \div 20$

7. $n \div 37 = 3$

8. $1106 \div 79$

11. $512 \div 8$

13. $n \div 2^3 = 101$

15. $n \div 18 = 18$

16. $473 \div 11$

17. $3627 \div 9$

18. $n \div 97 = 100$

19. $n \div 15 = 214$

20. $n \div 9 = 83$

21. The remainder for $2408 \div 97$

26. $n \div 13 = 7$

28. $1{,}234{,}567{,}890 \div 123{,}456{,}789$

Division with Remainders

The remainder for each division problem is the divisor for the next problem in that row. The last problem in the row should have 0 as its remainder.

1. A
$$\begin{array}{r} 43 \\ 68\overline{)2987} \\ 272 \\ \hline 267 \\ 204 \\ \hline 63 \end{array}$$

B
$$\begin{array}{r} 115 \\ 63\overline{)7290} \\ 63 \\ \hline 99 \\ 63 \\ \hline 360 \\ 315 \\ \hline 45 \end{array}$$

C
$$\begin{array}{r} 177 \\ 45\overline{)8002} \\ 45 \\ \hline 350 \\ 315 \\ \hline 352 \\ 315 \\ \hline 37 \end{array}$$

D
$$\begin{array}{r} 51 \\ 37\overline{)1887} \\ 185 \\ \hline 37 \\ 37 \\ \hline 0 \end{array}$$

2. A
$$\begin{array}{r} 15 \\ 81\overline{)1272} \\ 81 \\ \hline 462 \\ 405 \\ \hline 57 \end{array}$$

B
$$\begin{array}{r} 12 \\ 57\overline{)689} \\ 57 \\ \hline 119 \\ 114 \\ \hline 5 \end{array}$$

C
$$\begin{array}{r} 656 \\ 5\overline{)3282} \\ 30 \\ \hline 28 \\ 25 \\ \hline 32 \\ 30 \\ \hline 2 \end{array}$$

D
$$\begin{array}{r} 472 \\ 2\overline{)944} \\ 8 \\ \hline 14 \\ 14 \\ \hline 4 \\ 4 \\ \hline 0 \end{array}$$

3. Make this division problem so the remainder is zero. answers will vary

$\overline{)822}$

divisors: 1, 2, 6, 137, 411, 822

4. Find a 2-digit divisor so the remainder is zero. answers will vary

$\overline{)432}$

12, 16, 18, 24, 27, 36, 48, 54, 72

5. Complete this division problem so the remainder is 7.

$$\begin{array}{r} 15 \\ 14\overline{)217} \\ 210 \\ \hline 7 \end{array}$$

6. Complete this division problem so the remainder is 11.

$$\begin{array}{r} 13 \\ 13\overline{)180} \\ 13 \\ \hline 50 \\ 39 \\ \hline 11 \end{array}$$

Answers may vary in exercises 3 through 6

Square Roots of Positive Numbers

1. Give the number for **x** in each equation.

A $x^2 = 16$

$x = 4$

B $x^2 = 100$

$x = 10$

C $x^2 = 81$

$x = 9$

2. Since $4^2 = 16$ we say that the **square root** of 16 is equal to 4. We write $\sqrt{16} = 4$. Give the missing number.

A $\sqrt{25} =$ 5

B $\sqrt{64} =$ 8

C $\sqrt{36} =$ 6

D $\sqrt{1} =$ 1

E $\sqrt{49} =$ 7

F $\sqrt{100} =$ 10

3. Give the missing number.

A $\sqrt{144} = 12$

B $\sqrt{81} = 9$

C $\sqrt{16} = 4$

D $\sqrt{400} = 20$

E $\sqrt{\frac{1}{4}} = \frac{1}{2}$

F $\sqrt{\frac{1}{100}} = \frac{1}{10}$

4. Give the missing number for each part.

A $13 \times 13 =$ 169

$\sqrt{169} = 13$

B $27 \times 27 =$ 729

$\sqrt{729} = 27$

C $101 \times 101 =$ 10,201

$\sqrt{10{,}201} = 101$

D $21 \times 21 =$ 441

$\sqrt{441} = 21$

E $17 \times 17 =$ 289

$\sqrt{289} = 17$

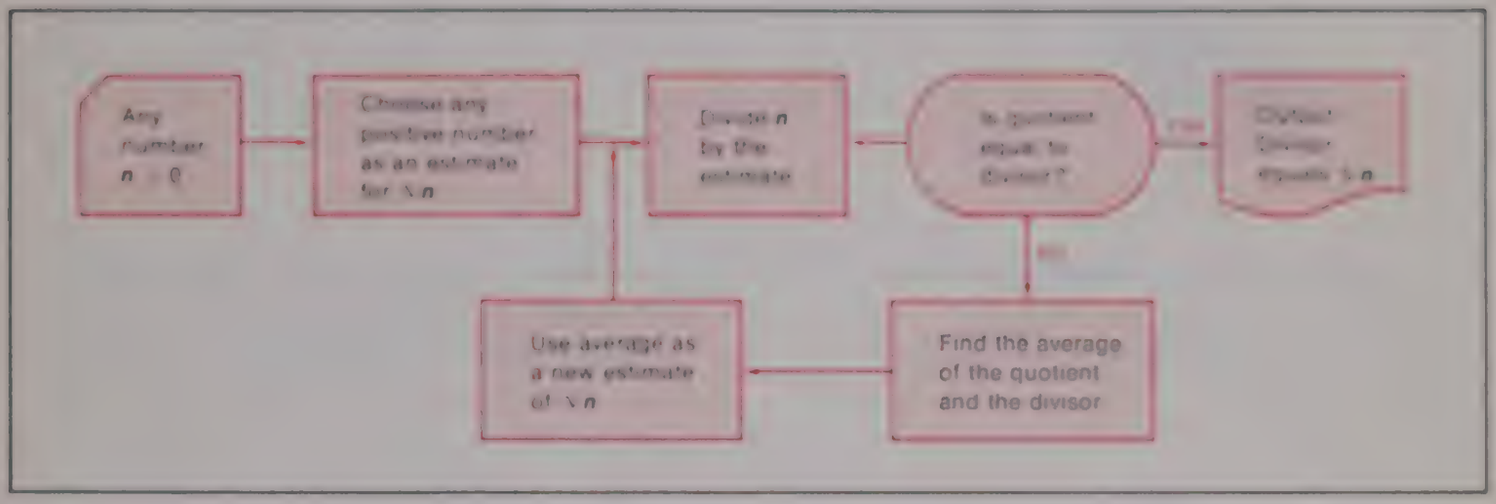
F $68 \times 68 =$ 4624

$\sqrt{4624} = 68$

Finding Square Roots by Dividing

The positive **square root** of 9 is 3 because $9 = 3 \times 3$.
We write $\sqrt{9} = 3$. This is read "The square root of 9 is 3."

The flow chart shows instructions for finding the positive square root of a number to any desired accuracy.



Example: Find $\sqrt{4489}$.

A Estimate: Since $60^2 = 3600$ and $70^2 = 4900$, $\sqrt{4489} \approx 65$

B
$$\begin{array}{r} 6\ 9 \\ 65\overline{)4489} \\ 390 \\ \hline 589 \\ 585 \\ \hline 4 \end{array}$$

C Average of 65 and 69 is 67

D
$$\begin{array}{r} 6\ 7 \\ 67\overline{)4489} \\ 402 \\ \hline 469 \\ 469 \\ \hline \end{array}$$

E Since $67 \times 67 = 4489$, $\sqrt{4489} = 67$

Use the method above to find these square roots.

1. $\sqrt{1089}$ 33

2. $\sqrt{1849}$ 43

3. $\sqrt{6241}$ 79

4. $\sqrt{8464}$ 92

5. $\sqrt{3025}$ 55

6. $\sqrt{9801}$ 99

7. $\sqrt{10,816}$ 104

8. $\sqrt{841}$ 29

Help the children to interpret the flow chart at the top of the page. You may want to provide another example for them to follow. Use only numbers that are perfect squares.

Money Problems

1. Janet bought 12 cans of dog food for her dog. The food cost 23 cents per can. How much did she pay for the dog food? $2.76

2. A case of canned vegetables sells for $7.44. If there are 24 cans in a case, how much is the cost per can? 31¢

3. Dave bought a car priced at $4170. He traded his old car on the new one and received $1200 off on the price of the new car. He paid the remainder in 18 equal payments. How much was each payment? $165

4. The Treasury Department announced the amount of money in circulation in the U.S. was $58,383,190,429. If the population of the United States is 207,000,000, how much money per person is this? Round your answer to the nearest dollar. $282

5. Peggy had the same number of quarters as pennies. Altogether she had $1.82. How many quarters did she have? 7

6. Jack had as many nickels as pennies. Altogether he had $1.38. How many of each coin did Jack have? 23
trial and error solution

7. If ball point pens cost $.69 each, what is the largest number that could be bought for $20.00? How much money would be left?
28 pens 68¢ remaining

8. Meg had $14.60. She spent half of this for a sweater and half of what she had left for gloves. She also paid 50¢ for bus fare. How much did she have left? $3.15

Abundant, Deficient, and Perfect Numbers

1. Study the first three examples in the table below to understand what is meant by **abundant**, **deficient**, and **perfect** numbers. Then complete the table.

	n	Factors of n	Sum of Factors except n	Comparison of sum and n	Abundant number	Deficient number	Perfect number
A	12	1, 2, 3, 4, 6, 12	1 + 2 + 3 + 4 + 6 = 16	16 > 12	✓		
B	10	1, 2, 5, 10	1 + 2 + 5 = 8	8 < 10		✓	
C	6	1, 2, 3, 6	1 + 2 + 3 = 6	6 = 6			✓
D	20	1,2,4,5,10,20	1+2+4+5+10=22	22 > 20	✓		
E	8	1,2,4,8	1+2+4 =7	7 < 8		✓	
F	16	1,2,4,8,16	1+2+4+8=15	15 < 16		✓	
G	18	1,2,3,6,9,18	1+2+3+6+9=21	21 > 18	✓		
H	24	1,2,3,4,6,8,12,24	36	36 > 24	✓		
I	28	1,2,4,7,14,28	28	28 = 28			✓
J	30	1,2,3,5,6,10,15, 30	42	42 > 30	✓		
K	45	1,3,5,9,15,45	33	33 < 45		✓	
L	100	1,2,4,5,10,20, 25,50, 100	117	117 > 100	✓		
M	111	1,3,37,111	41	41 < 111		✓	
N	200	1,2,4,5,8,10,20 25,40,50,100,200	265	265 > 200	✓		
O	80	1,2,4,5,8,10,16, 20,40,80	106	106 > 80	✓		
P	64	1,2,4,8,16,32,64	63	63 < 64		✓	
Q	63	1,3,7,9,21,63	41	41 < 63		✓	

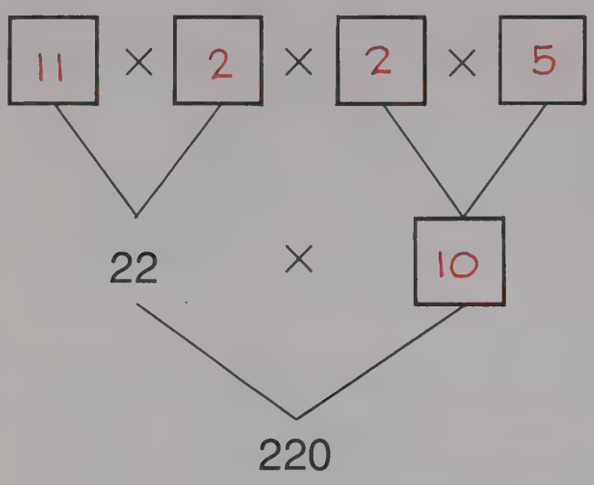
2. A Find all the factors of 496.
1,2,4,8,16,31,62,124,248, 496

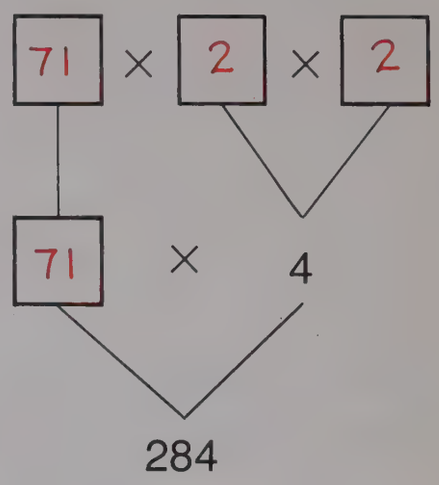
B Show that 496 is a perfect number.
1+2+4+8+16+31+62+124+248= 496

Only a few perfect numbers have been found. The next perfect number larger than 496 is 8128.

Amicable Numbers

1. Complete the Factor Trees.





2. Find all the factors of 220.

1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, 220

3. Find the sum of all the factors of 220, except 220.

284

4. Find all the factors of 284.

1, 2, 4, 71, 142, 284

5. Find the sum of all the factors of 284, except 284.

220

6. The pair of numbers 220 and 284 are called **amicable** or **friendly numbers.** Can you see why they are given that name?

The sum of the factors except the number equals the other number.

7. Look in a reference book to find some other pairs of amicable numbers. Check their factors to see why they are amicable.

There are only 5 pairs of amicable numbers that are less than ten thousand. They are: 220 and 284; 1184 and 1210; 2620 and 2924; 5020 and 5564; 6232 and 6368.

Prime and Composite Numbers

1. Express each even number as the sum of two **prime numbers**. There may be more than one way to do this for some numbers.

A $8 = 3 + 5$

B $12 = 5 + 7$

C $20 = 17 + 3$

D $16 = 13 + 3$

E $10 = 7 + 3$

F $24 = 19 + 5$

G $30 = 23 + 7$

H $36 = 31 + 5$

I $50 = 47 + 3$

J $100 = 97 + 3$

2. Express each even number as the sum of **two odd composite** numbers.

A $18 = 9 + 9$

B $24 = 9 + 15$

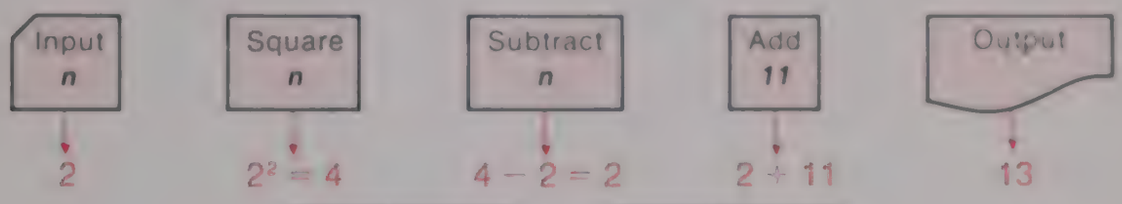
C $30 = 9 + 21$

D $34 = 9 + 25$

E $36 = 21 + 15$

F $40 = 15 + 25$

3. Use the flow chart below to complete the Input-Output tables.



Input	Output
0	11
1	11
2	13
3	17
4	23
5	31

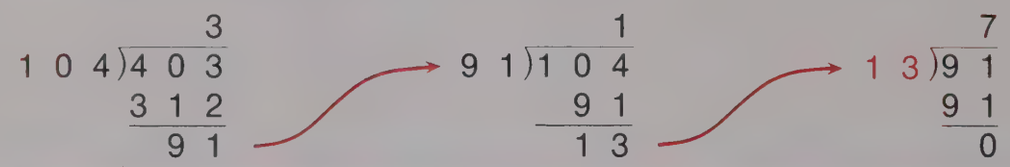
Input	Output
6	41
7	53
8	67
9	83
10	101
11	121

4. Which output numbers in Exercise 3 are prime numbers? Which are composite?
All are prime except 121.

Greatest Common Factor

You can find the greatest common factor of two large numbers by using a division method called the **Euclidean Algorithm.** Study the example below to see the steps of this method.

EXAMPLE: Find the GCF of 104 and 403.



The divisor that gives a 0 remainder is the GCF of the two numbers.

GCF of 104 and 403 is 13.

$104 = 13 \times 8$, $403 = 13 \times 31$

Find the greatest common factor of each pair of numbers.

1. 63 and 196

7

2. 435 and 1189

29

3. 264 and 1608

24

4. 342 and 900

18

5. 6666 and 8383

101

6. 25,787 and 253,291

241

You may need to provide additional examples for some children. For example, show that 14 is the GCF of 56 and 98.

A Divisibility Rule for 9

A number is divisible by 9 (zero remainder) if the sum of the digits of the number is a multiple of 9.

EXAMPLE 1: 7345 ⟶ 7 + 3 + 4 + 5 = 18

Since 18 is a multiple of 9, 7345 is divisible by 9.

EXAMPLE 2: 5794 ⟶ 5 + 7 + 9 + 4 = 25

Since 25 is not a multiple of 9, 5794 is not divisible by 9.

1. Complete the table below.

n	Sum of digits	Is *n* divisible by 9?	*n*	Sum of digits	Is *n* divisible by 9?
126	9	yes	93,572	26	no
747	18	yes	81,933	24	no
3996	27	yes	70,002	9	yes
8138	20	no	51,643	19	no
5778	27	yes	22,221	9	yes
17,532	18	yes	234,567	27	yes
64,281	21	no	568,944	36	yes

2. What year were you born? Is the number for that year divisible by 9? answers will vary

3. ▲ Is the present year number divisible by 9?

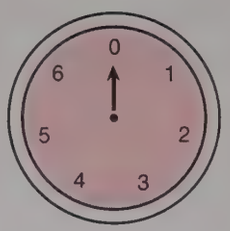
   ■ What is the next year that will be divisible by 9? 1980, 1989, 1998 are divisible by 9

4. ▲ Write any numeral that uses all ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 exactly once. Is the number represented divisible by 9? yes

   ■ Try it again. What can you conclude about all such numerals?
   Always divisible by 9 because the sum of all the digits will be 45 no matter what order they are written.

Seven-Clock Arithmetic

Think of a clock face with just seven numerals on its face as shown. You can do seven-clock arithmetic by thinking about the clock face.



1. Answer the question. Then solve the equation.

A What time will it be 3 hours after 5 o'clock? 1 o'clock

$5 + 3 = n$, $n =$ 1

B What time will it be 4 hours before 2 o'clock? 5 o'clock

$2 - 4 = n$, $n =$ 5

2. Solve these equations using seven-clock arithmetic.

A $2 + 4 = x$, $x =$ 6

B $4 + 4 = y$, $y =$ 1

C $5 - 4 = q$, $q =$ 1

D $6 + 6 = p$, $p =$ 5

E $1 - 4 = t$, $t =$ 4

F $4 - 6 = u$, $u =$ 5

3. Find the sum, then find the product. Use seven-clock numbers.

A $4 + 4 + 4 =$ 5, $3 \times 4 =$ 5

B $6 + 6 =$ 5, $2 \times 6 =$ 5

C $2 + 2 + 2 + 2 =$ 1, $4 \times 2 =$ 1

4. Find the products.

A $2 \times 5 =$ 3

B $4 \times 2 =$ 1

C $5 \times 3 =$ 1

D $6 \times 4 =$ 3

E $3 \times 3 =$ 2

F $4 \times 5 =$ 6

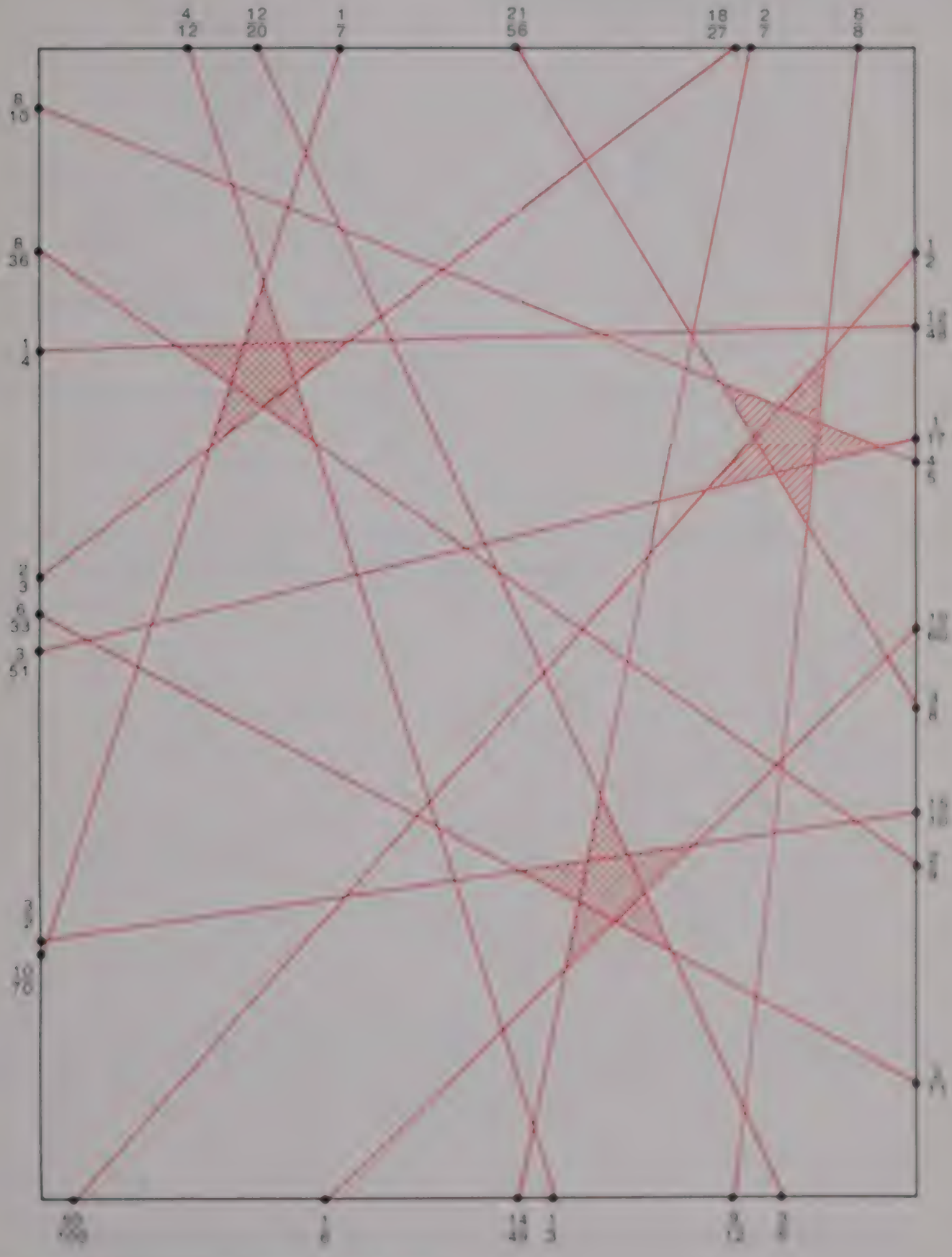
5. Complete the addition and multiplication tables using seven-clock arithmetic.

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

Fraction Stars

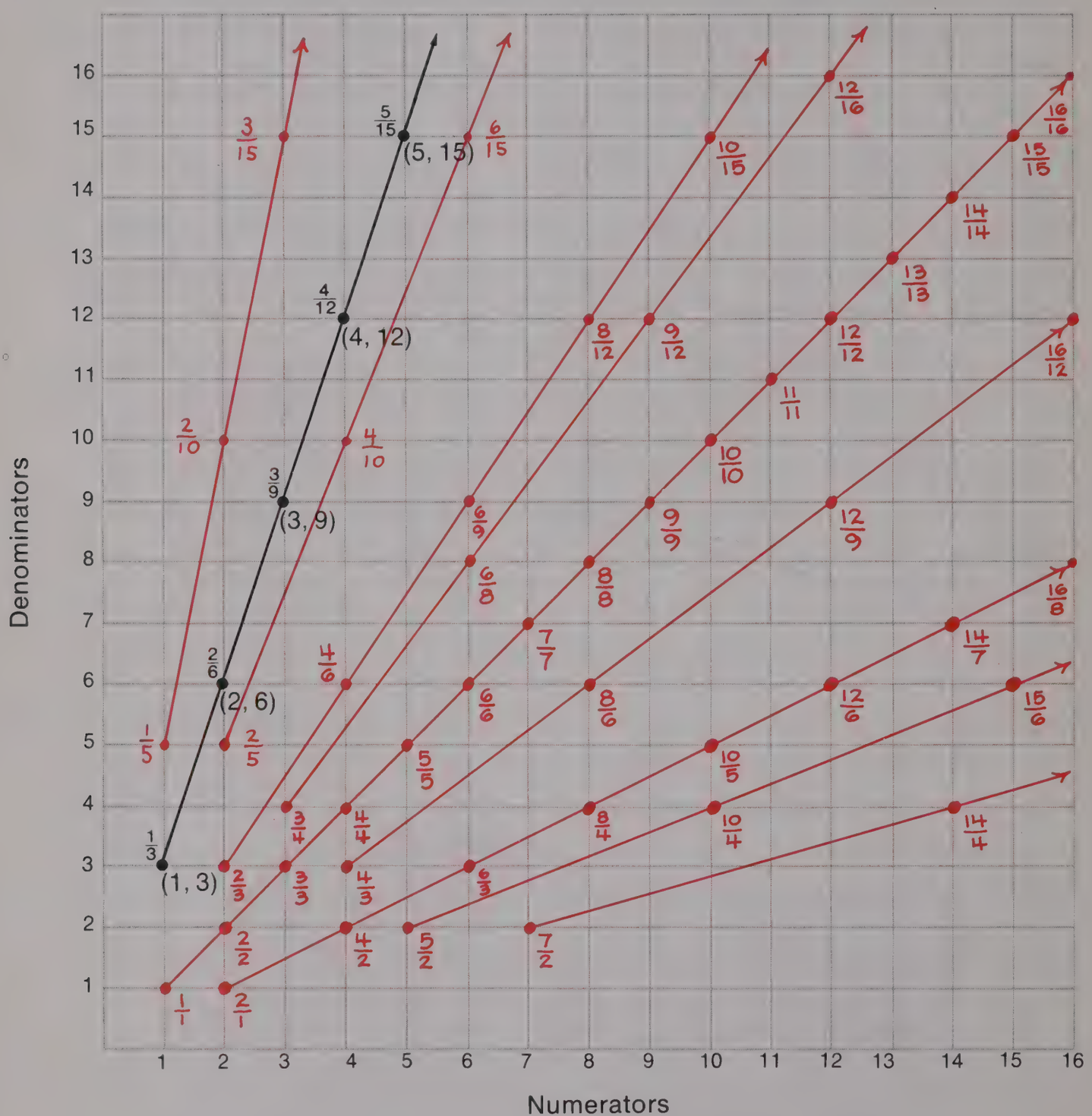
Connect the points for pairs of equivalent fractions with segments.
If you do this correctly you should be able to find 3
five-pointed stars in your drawing.



Children will need rulers for the activity on this page.

Equivalent Fraction Rays

The coordinates of the points on the ray form a set of equivalent fractions. Can you construct "equivalent fraction rays" for each fraction named below? Name the fraction for each point on the ray for the pairs of whole numbers that are the coordinates of that point.



1. $\frac{2}{3}$ **2.** $\frac{1}{5}$ **3.** $\frac{5}{2}$ **4.** $\frac{3}{4}$ **5.** $\frac{2}{1}$ **6.** $\frac{2}{5}$ **7.** $\frac{7}{2}$ **8.** $\frac{4}{3}$ **9.** $\frac{1}{1}$

10. Can you explain why no two "equivalent fraction rays" will ever intersect?

No fraction can belong to more than one set of equivalent fractions.

Finding Equivalent Fractions

1. Think of each fraction given as being in a set of equivalent fractions. Give the missing numerator and denominator.

A $\frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots$

B $\frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \ldots$

C $\frac{9}{30}, \frac{12}{40}, \frac{15}{50}, \ldots$

D $\frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \ldots$

E $\frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \ldots$

F $\frac{35}{42}, \frac{40}{48}, \frac{45}{54}, \ldots$

G $\frac{14}{21}, \frac{16}{24}, \frac{18}{27}, \ldots$

H $\frac{63}{90}, \frac{70}{100}, \frac{77}{110}, \ldots$

2. Give the lowest-terms fraction for each set of fractions above.

A $\frac{3}{4}$ B $\frac{2}{3}$ C $\frac{3}{10}$ D $\frac{3}{5}$ E $\frac{5}{8}$ F $\frac{5}{6}$ G $\frac{2}{3}$ H $\frac{7}{10}$

3. Think of each fraction given as being in a set of equivalent fractions. Find the fractions on each side of the given fraction.

A $\frac{18}{20}, \frac{27}{30}, \frac{36}{40}, \ldots$

B $\frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots$

C $\frac{35}{45}, \frac{42}{54}, \frac{49}{63}, \ldots$

D $\frac{21}{27}, \frac{28}{36}, \frac{35}{45}, \ldots$

E $\frac{16}{27}, \frac{32}{54}, \frac{48}{81}, \ldots$

F $\frac{17}{18}, \frac{34}{36}, \frac{51}{54}, \ldots$

G $\frac{64}{72}, \frac{72}{81}, \frac{80}{90}, \ldots$

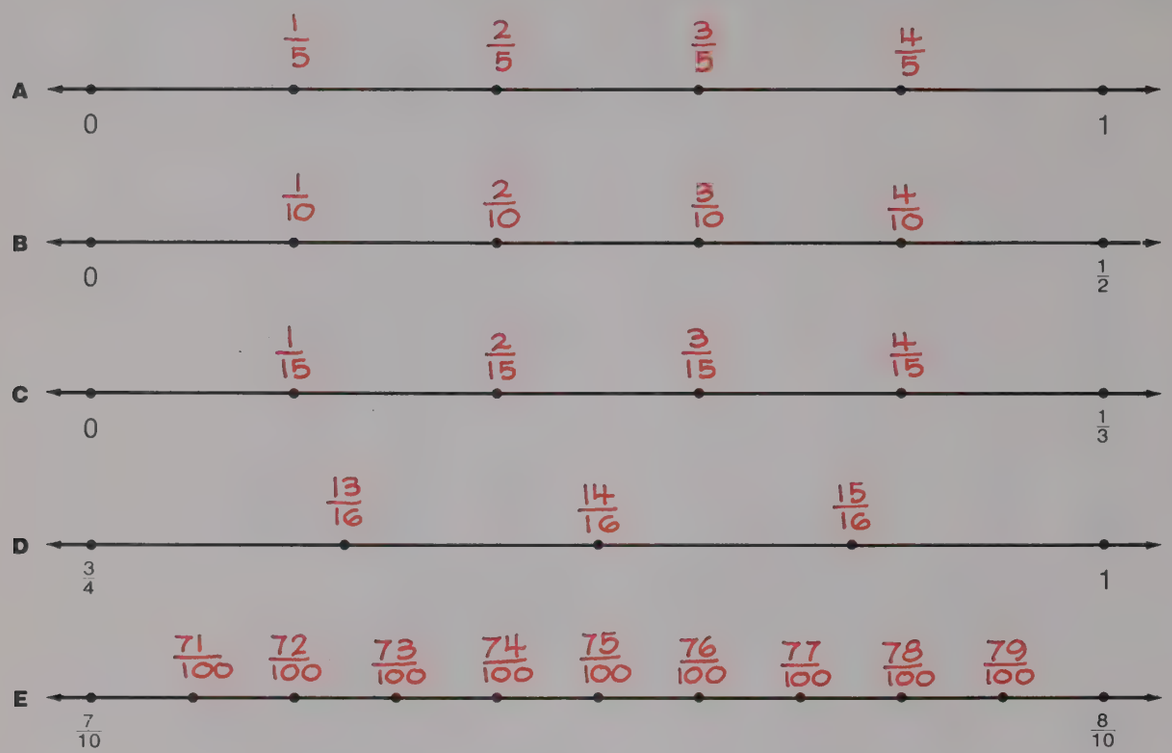
H $\frac{10}{26}, \frac{12}{30}, \frac{14}{35}, \ldots$

4. Check your answers to Exercise 3 by reducing each of the three fractions to lowest terms. Each of the three fractions should reduce to the same fraction.

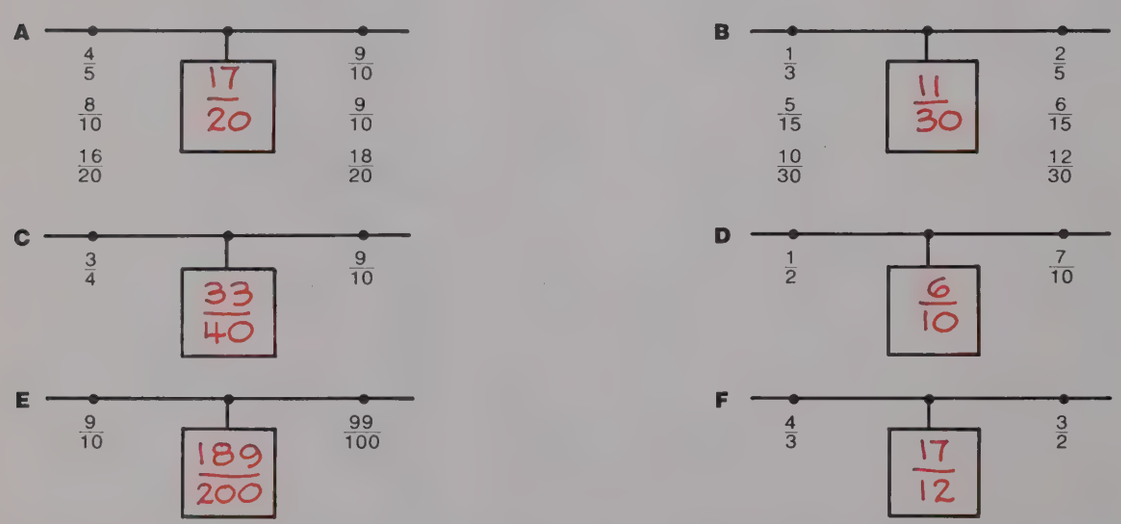
A. $\frac{9}{10}$ B. $\frac{5}{8}$ C. $\frac{7}{9}$ D. $\frac{7}{9}$ E. $\frac{16}{27}$ F. $\frac{17}{18}$ G. $\frac{8}{9}$ H. $\frac{2}{5}$

Fractional Numbers

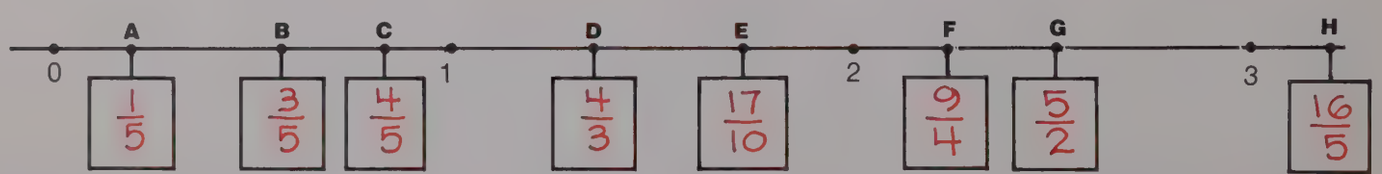
1. Write a fraction for each point shown on the number line.



2. Give the fractional number for the point halfway between the two given points. Think of equivalent fractions for the points.



3. Estimate the fractional number for each lettered point. Write the fraction for the number on the tag. Answers will vary



Fractional Number Problems

Each problem can be solved by writing an equation about a pair of equivalent fractions. Write and solve an equation for each problem.

1. There are 36 students in one classroom. 2 out of every 9 students wear contact lenses. How many wear contact lenses?

   Solution:
   n = number of students who wear contact lenses.

   $\frac{2}{9} = \frac{n}{36}$, $n =$ 8

2. There are 85 students in the sixth grade. 3 out of 5 ride a bus to school. How many ride the bus?

   Solution:
   n = number of students riding on buses.

   $\frac{3}{5} = \frac{n}{85}$, $n =$ 51

3. Janet is $\frac{3}{4}$ as tall as her father. Her father is 188 centimeters tall. How tall is Janet?

   Solution:
   n = Janet's height.

   $\frac{3}{4} = \frac{n}{188}$ $n = 141$

4. Tom shot 100 free throws. He made 2 out of every 5 shots. How many free throws did Tom make?

   Solution:

   $\frac{2}{5} = \frac{n}{100}$ $n = 40$

5. Debbie made 2 hits in every 3 times at bat in softball. Altogether she made 8 hits. How many times did she bat?

   Solution:

   $\frac{2}{3} = \frac{8}{n}$ $n = 12$

6. Five cents sales tax must be paid on each dollar of a purchase. How much sales tax on a $12 sweater?

   Solution:

   $\frac{5}{100} = \frac{n}{1200}$ $n = 60$

7. Pam plans to save 3 dollars out of every 5 dollars she earns. If she earns $40 how much will she plan to save?

   Solution

   $\frac{3}{5} = \frac{n}{40}$ $n = 24$

8. Dave pays $\frac{3}{10}$ of his salary for rent. He earns $600 a month. What does he pay for rent?

   Solution

   $\frac{3}{10} = \frac{n}{600}$ $n = 180$

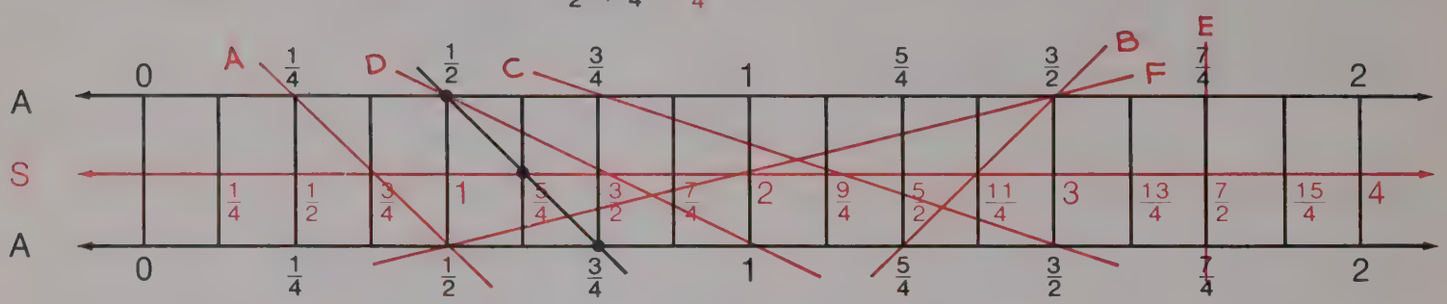
Children should solve the equations by thinking about equivalent fractions since multiplication and division of fractional numbers will be studied later.

Adding with a Nomograph

1. The slanted line drawn across the three number lines of the **nomograph** shows the addition equation.

A A S

$\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$



Connect the two addends with a straight line, then find the sum and complete the equation.

A A S

A $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

B $\frac{3}{2} + \frac{5}{4} = \frac{11}{4}$

C $\frac{3}{4} + \frac{3}{2} = \frac{9}{4}$

D $1 + \frac{1}{2} = \frac{3}{2}$

E $\frac{7}{4} + \frac{7}{4} = \frac{7}{2}$

F $\frac{3}{2} + \frac{1}{2} = 2$

2. Find the missing addend in each equation. Use the nomograph if you need help.

A $\frac{1}{2} + \boxed{\frac{3}{4}} = \frac{5}{4}$

B $1 + \boxed{\frac{3}{2}} = \frac{5}{2}$

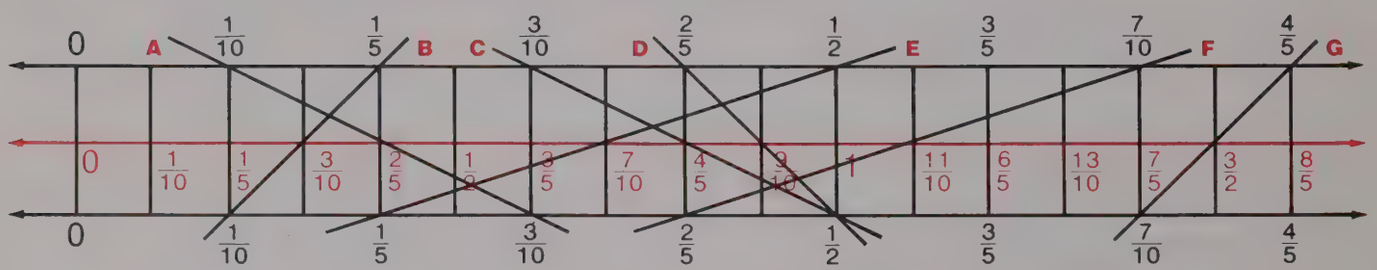
C $\boxed{\frac{1}{2}} + \frac{5}{4} = \frac{7}{4}$

D $\boxed{\frac{5}{4}} + \frac{7}{4} = 3$

E $\frac{3}{4} + \boxed{\frac{3}{2}} = \frac{9}{4}$

F $\frac{7}{4} + \boxed{\frac{7}{4}} = \frac{7}{2}$

3. Write an addition equation for each lettered line on the nomograph.



A $\frac{1}{10} + \frac{3}{10} = \frac{2}{5}$

B $\frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

C $\frac{3}{10} + \frac{1}{2} = \frac{4}{5}$

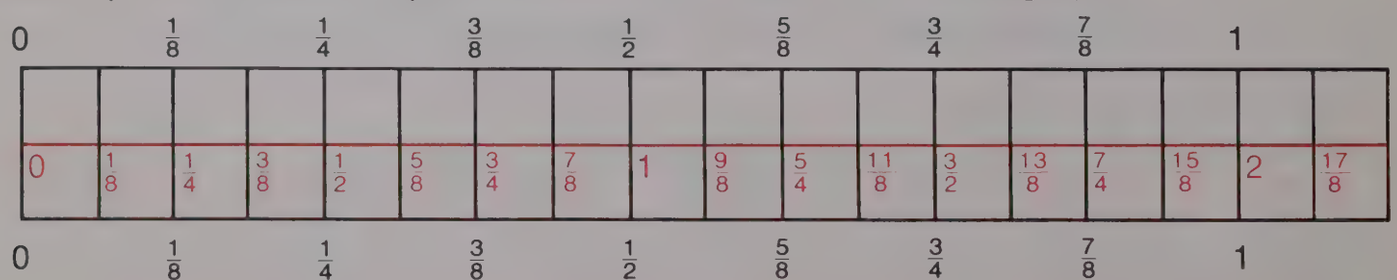
D $\frac{2}{5} + \frac{1}{2} = \frac{9}{10}$

E $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$

F $\frac{7}{10} + \frac{2}{5} = \frac{11}{10}$

G $\frac{4}{5} + \frac{7}{10} = \frac{3}{2}$

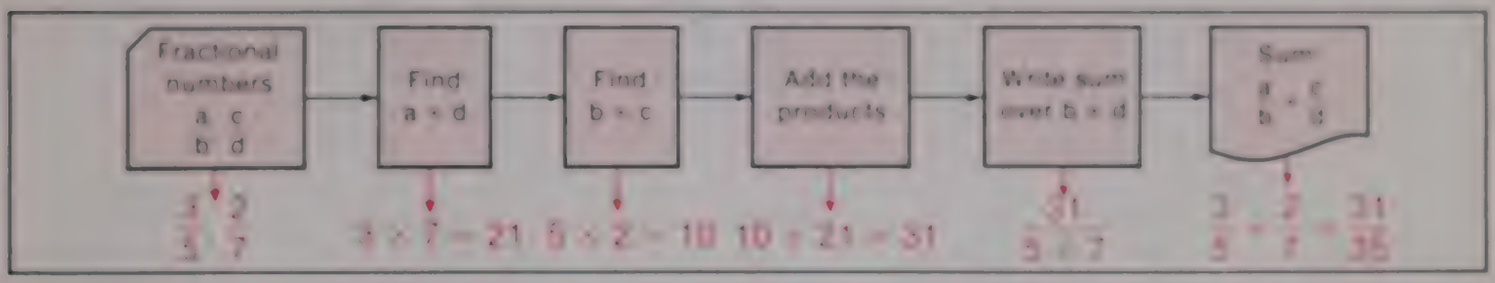
4. Make up some addition equations and show them on this nomograph.



Some children may be encouraged to make up their own nomographs. Note that the units on the scale for the sum are only half as long as the units on the scales for the addends.

Adding and Subtracting Fractional Numbers

Study the flow chart to learn a shortcut method of adding any two fractional numbers



Follow the flow chart to find the sums below.

1. $\frac{1}{2} + \frac{2}{3}$ $\frac{1 \times 3 + 2 \times 2}{2 \times 3} = \frac{7}{6}$

2. $\frac{3}{10} + \frac{5}{6}$ $\frac{3 \times 6 + 5 \times 10}{6 \times 10} = \frac{68}{60} = \frac{17}{15}$

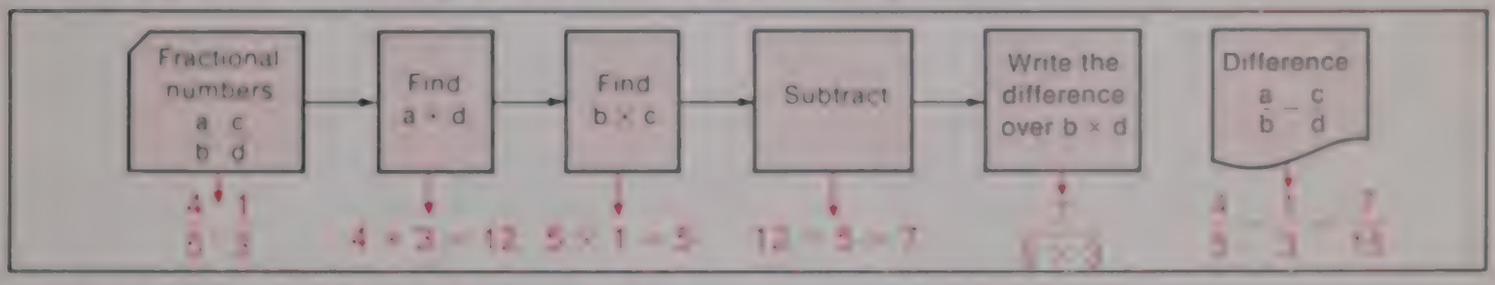
3. $\frac{3}{4} + \frac{9}{10}$ $\frac{3 \times 10 + 4 \times 9}{4 \times 10} = \frac{66}{40} = \frac{33}{20}$

4. $\frac{7}{3} + \frac{1}{7}$ $\frac{7 \times 7 + 1 \times 3}{3 \times 7} = \frac{52}{21}$

5. $\frac{5}{8} + \frac{2}{3}$ $\frac{5 \times 3 + 2 \times 8}{8 \times 3} = \frac{31}{24}$

6. $\frac{9}{10} + \frac{2}{11}$ $\frac{9 \times 11 + 2 \times 10}{10 \times 11} = \frac{119}{110}$

This is a subtraction flow chart for fractional numbers



Find the differences. Use the flow chart above.

7. $\frac{9}{10} - \frac{1}{3}$ $\frac{17}{30}$

8. $\frac{11}{6} - \frac{5}{8}$ $\frac{58}{48}$ or $\frac{29}{24}$

9. $\frac{11}{12} - \frac{4}{9}$ $\frac{51}{108}$ or $\frac{17}{36}$

10. $\frac{1}{1} - \frac{5}{7}$ $\frac{2}{7}$

11. $\frac{11}{25} - \frac{1}{4}$ $\frac{19}{160}$

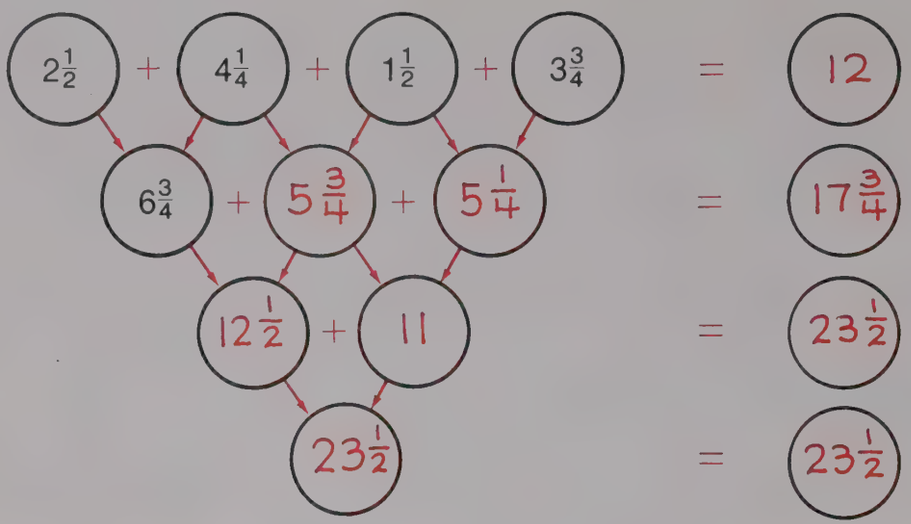
12. $\frac{10}{13} - \frac{8}{11}$ $\frac{6}{143}$

The two flow charts present the addition and subtraction algorithms for fractional numbers

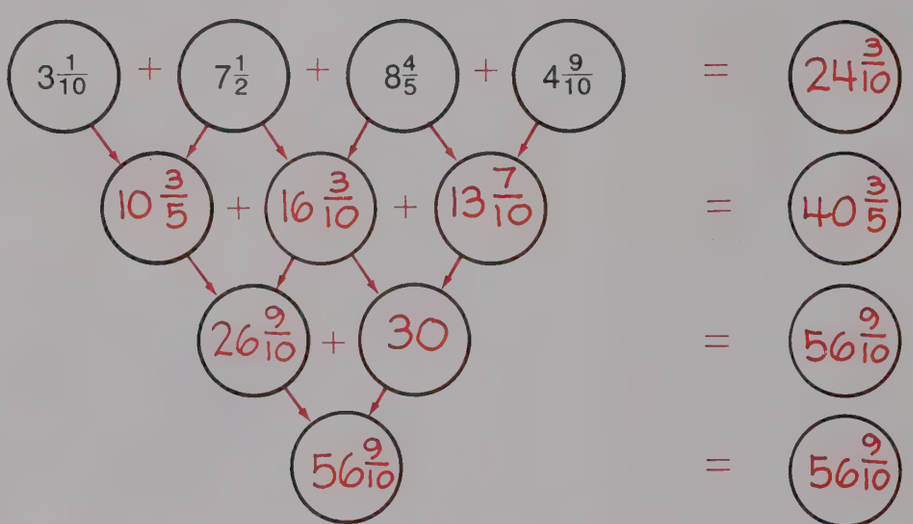
$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ and $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

Triangle Sums

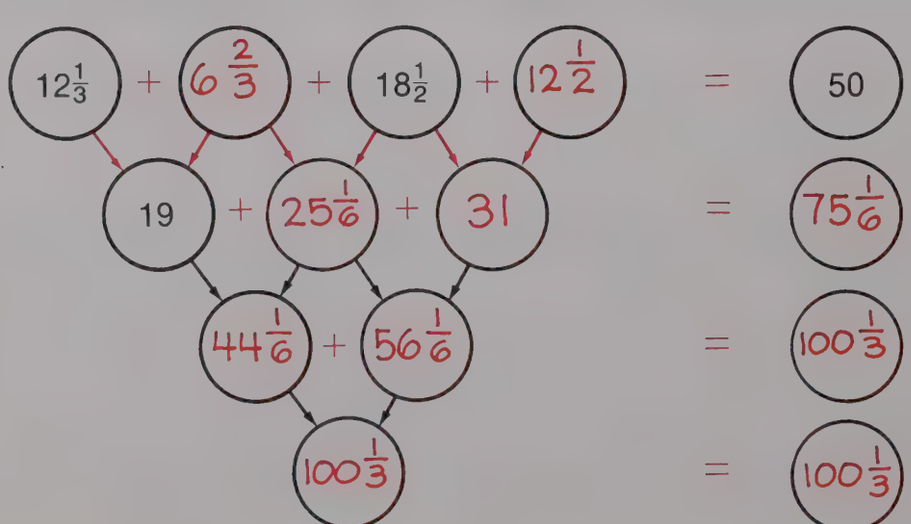
1. Find all the sums indicated.



2. Find all the sums indicated.



3. Complete the diagram. Find the missing addends and sums.

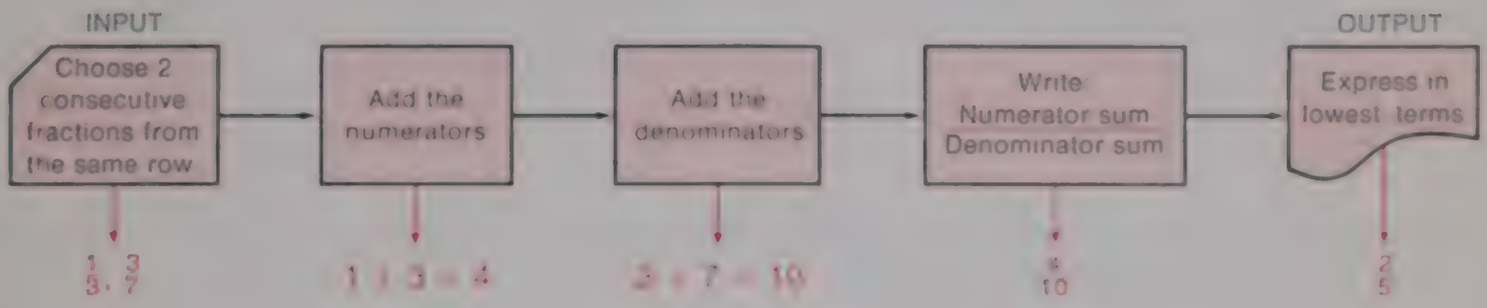


A Farey Sequence

The fractions in the two rows comprise an interesting sequence called a **Farey Sequence** Study the flow chart, then try the exercises.

Row 1→ $\frac{0}{1}$ $\frac{1}{6}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{1}{1}$

Row 2→ $\frac{1}{7}$ $\frac{1}{5}$ $\frac{2}{7}$ $\frac{2}{5}$ $\frac{1}{2}$ $\frac{3}{5}$ $\frac{5}{7}$ $\frac{4}{5}$ $\frac{6}{7}$



Use the flow chart to find the output number for these pairs.

1. $\frac{1}{2}, \frac{3}{5}$ $\frac{4}{7}$
2. $\frac{4}{7}, \frac{2}{3}$ $\frac{3}{5}$
3. $\frac{3}{7}, \frac{4}{7}$ $\frac{1}{2}$
4. $\frac{3}{4}, \frac{5}{6}$ $\frac{4}{5}$
5. $\frac{5}{6}, \frac{1}{1}$ $\frac{6}{7}$
6. $\frac{1}{4}, \frac{1}{3}$ $\frac{2}{7}$
7. $\frac{0}{1}, \frac{1}{6}$ $\frac{1}{7}$
8. $\frac{5}{7}, \frac{4}{5}$ $\frac{3}{4}$
9. $\frac{1}{5}, \frac{2}{7}$ $\frac{1}{4}$
10. $\frac{2}{3}, \frac{3}{4}$ $\frac{5}{7}$
11. $\frac{3}{5}, \frac{5}{7}$ $\frac{2}{3}$
12. $\frac{2}{7}, \frac{2}{5}$ $\frac{1}{3}$
13. $\frac{4}{5}, \frac{6}{7}$ $\frac{5}{6}$
14. $\frac{1}{7}, \frac{1}{5}$ $\frac{1}{6}$
15. $\frac{1}{6}, \frac{1}{4}$ $\frac{1}{5}$
16. $\frac{1}{3}, \frac{3}{7}$ $\frac{2}{5}$
17. $\frac{2}{5}, \frac{1}{2}$ $\frac{3}{7}$

18. What can you conclude about the location of each output number with respect to its input number for the sequence above?

The output number is between the two input numbers in the other row.

The flow chart presents a method sometimes called "Freshman Addition." It is used here to help the child discover some interesting things about Farey Sequences. Emphasize that fractional numbers are not added in this way.

Mixed Numerals, Improper Fractions

1. Suppose you wrote these numerals and symbols on cards.

Using three of the cards you could form a symbol for this improper fraction.

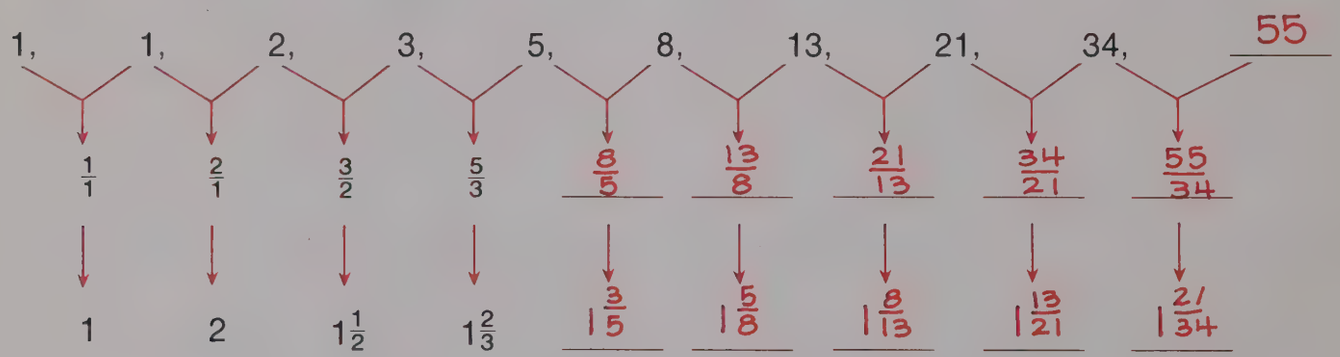
How many other symbols for improper fractions could you make from these cards using only 3 of them at a time?

$\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{8}{2}, \frac{11}{2}, \frac{5}{3}, \frac{7}{3}, \frac{8}{3}, \frac{11}{3}, \frac{7}{5}, \frac{8}{5}, \frac{11}{5}, \frac{8}{7}, \frac{11}{7}, \frac{11}{8}$

2. Write a mixed numeral or a whole number numeral for each of your improper fractions.

$1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4, 5\frac{1}{2}, 1\frac{2}{3}, 2\frac{1}{3}, 3\frac{2}{3}, 1\frac{2}{5}, 1\frac{3}{5}, 2\frac{1}{5}, 1\frac{1}{7}, 1\frac{4}{7}, 1\frac{3}{8}$

3. Study the pattern, then give the missing numbers in each row.



4. Complete each square below so that it is a magic square.

$2\frac{5}{6}$	$2\frac{1}{4}$	$2\frac{2}{3}$
$2\frac{5}{12}$	$2\frac{7}{12}$	$2\frac{3}{4}$
$2\frac{1}{2}$	$2\frac{11}{12}$	$2\frac{1}{3}$

$\frac{11}{5}$	$\frac{3}{2}$	2
$\frac{17}{10}$	$\frac{19}{10}$	$\frac{21}{10}$
$\frac{9}{5}$	$\frac{23}{10}$	$\frac{8}{5}$

The sequence of numbers in exercise 3 is the Fibonacci Sequence. Any pair of consecutive numbers in this sequence will form a lowest-terms fraction.

Graphing Functions

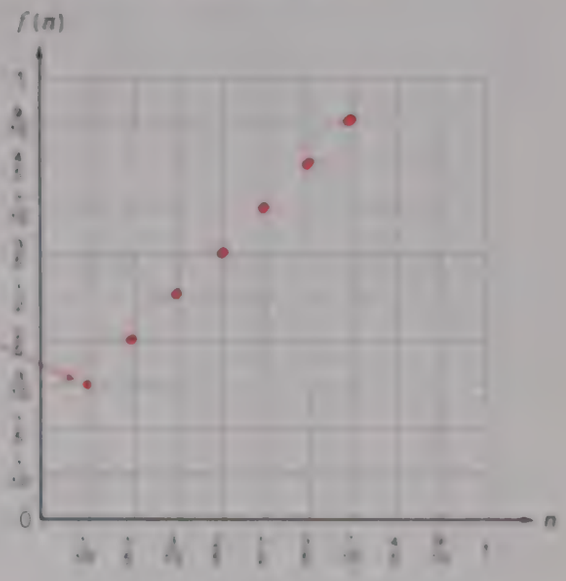
Complete the function table. then graph the number pairs on the grid.

1.

Function rule

$n \cdot \frac{1}{5}$

n	$f(n)$
$\frac{1}{10}$	$\frac{3}{10}$
$\frac{1}{5}$	$\frac{2}{5}$
$\frac{3}{10}$	$\frac{1}{2}$
$\frac{2}{5}$	$\frac{3}{5}$
$\frac{1}{2}$	$\frac{7}{10}$
$\frac{3}{5}$	$\frac{4}{5}$
$\frac{7}{10}$	$\frac{9}{10}$

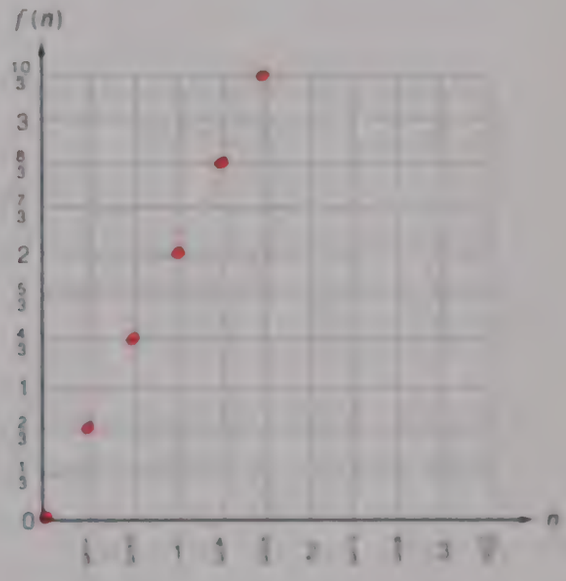


2.

Function rule

$n \cdot n$

n	$f(n)$
0	0
$\frac{1}{3}$	$\frac{2}{3}$
$\frac{2}{3}$	$\frac{4}{3}$
1	2
$\frac{4}{3}$	$\frac{8}{3}$
$\frac{5}{3}$	$\frac{10}{3}$

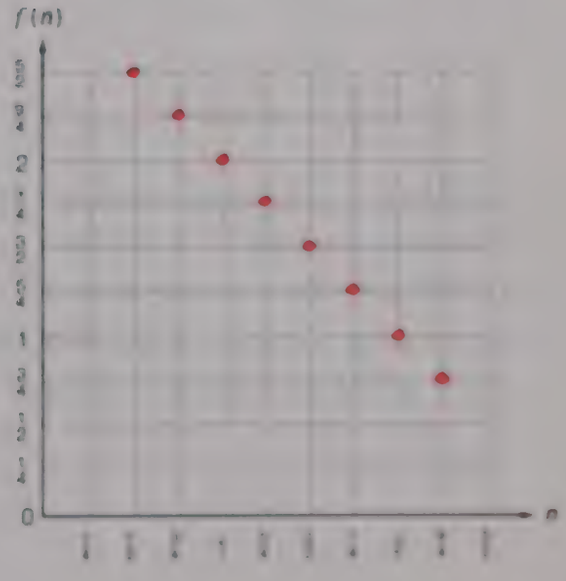


3.

Function rule

$3 \cdot n$

n	$f(n)$
$\frac{1}{2}$	$\frac{3}{2}$
$\frac{3}{4}$	$\frac{9}{4}$
1	2
$\frac{1}{4}$	$\frac{7}{4}$
$\frac{2}{3}$	$\frac{5}{3}$
$\frac{1}{4}$	$\frac{3}{4}$
2	1
$\frac{9}{4}$	$\frac{3}{4}$



Swimming Records

Work Space

1. Mark Spitz swam 100 meters freestyle in a record time of $51\frac{2}{10}$ seconds. How long would it have taken him to swim 200 meters at this rate? Give your answer in minutes and seconds.

1 minute $42\frac{4}{10}$ seconds

2. Mark Spitz's record for the 200 meter freestyle swim is 1 minute $52\frac{8}{10}$ seconds. How much more time is this than the answer to Exercise 1?

$10\frac{4}{10}$ seconds

3. Shane Gould swam 100 meters freestyle in $58\frac{1}{2}$ seconds. How much slower is this than Mark Spitz's record for the 100 meter race?

$7\frac{3}{10}$ seconds

4. Mark Spitz swam 100 meters using the butterfly stroke in $54\frac{3}{10}$ seconds. Mayumi Aoki set a women's record time of 1 minute $3\frac{3}{10}$ seconds for the 100 meter butterfly swim. How much slower was her time for the 100 meter race?

9 seconds

5. Brad Cooper had a time of 8 minutes $23\frac{4}{5}$ seconds in the men's 800 meter freestyle swim race. Kenna Rothhammer's time the women's 800 meter freestyle was $9\frac{9}{10}$ seconds more than Cooper's time. What was Rothhammer's time?

8 minutes $33\frac{7}{10}$ seconds

6. Shane Gould's time for the women's 1500 meter freestyle swim was 17 minutes $\frac{6}{10}$ seconds. Mike Burton's time for the same distance was 1 minute $8\frac{2}{100}$ seconds less than this. What was Burton's time?

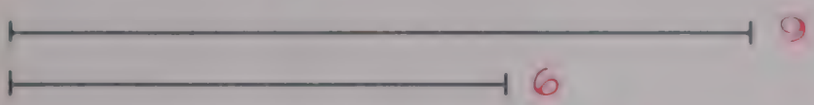
15 minutes $52\frac{58}{100}$ seconds

Mixed Numerals, Improper Fractions

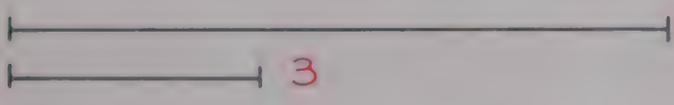
Measure each segment with a centimeter ruler. Then write an improper fraction that compares the length of the longer segment to the shorter one. Finally, give the mixed numeral that tells how many shorter segments it would take to match the longer segment.

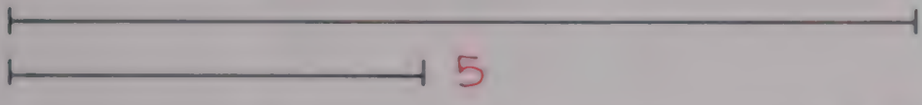
1. 

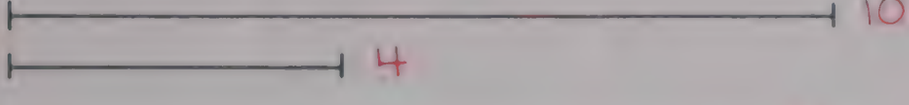
2. 

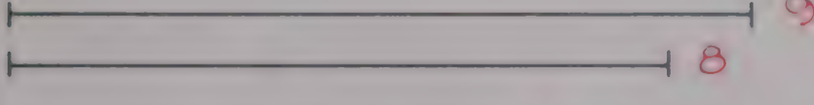
3. 

4. 

5. 

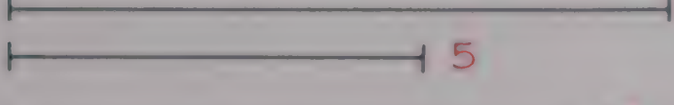
6. 

7. 

8. 

9. 

10. 

11. 

12. 

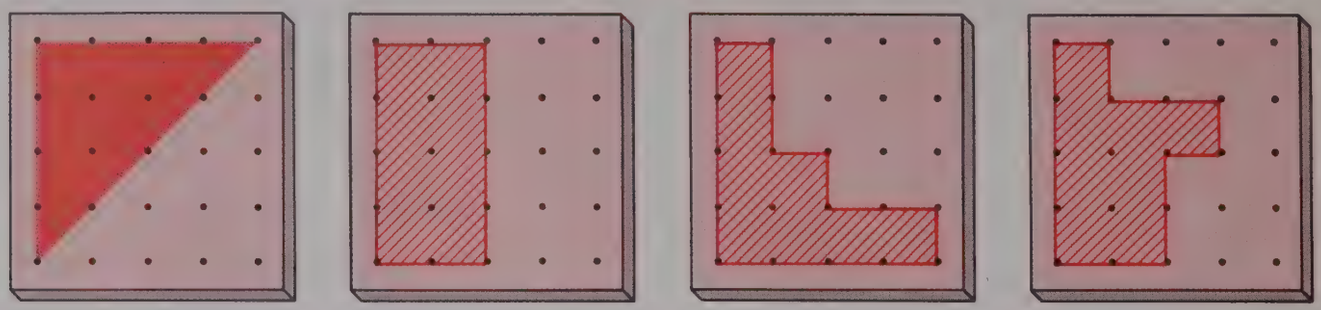
	Improper fraction	Mixed numeral
1.	$\frac{7}{2}$	$3\frac{1}{2}$
2.	$\frac{10}{3}$	$3\frac{1}{3}$
3.	$\frac{9}{6}$	$1\frac{1}{2}$
4.	$\frac{7}{4}$	$1\frac{3}{4}$
5.	$\frac{8}{3}$	$2\frac{2}{3}$
6.	$\frac{11}{5}$	$2\frac{1}{5}$
7.	$\frac{10}{4}$	$2\frac{1}{2}$
8.	$\frac{9}{8}$	$1\frac{1}{8}$
9.	$\frac{6}{4}$	$1\frac{1}{2}$
10.	$\frac{10}{8}$	$1\frac{1}{4}$
11.	$\frac{8}{5}$	$1\frac{3}{5}$
12.	$\frac{7}{1\frac{1}{2}}$	4

Centimeter rulers should be available to students for this activity

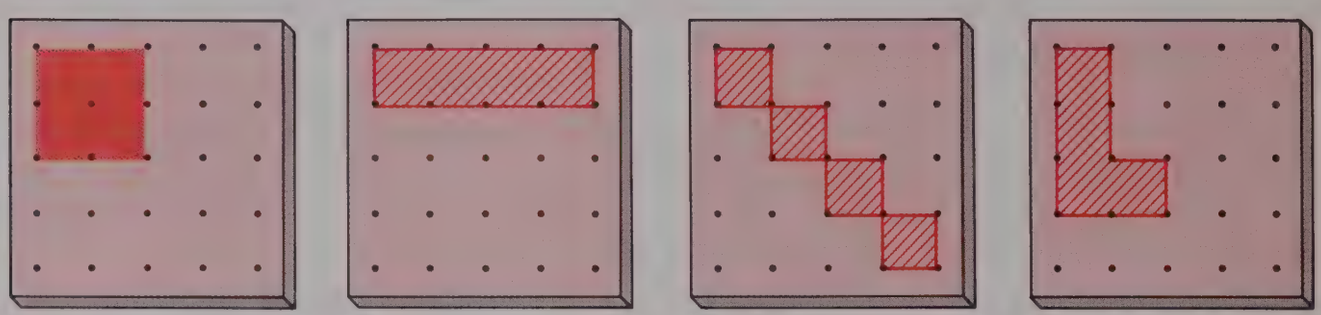
Exploring Unit Fractions

Can you show at least three different shapes for each given unit fraction?
One example is given for each fraction. Answers will vary

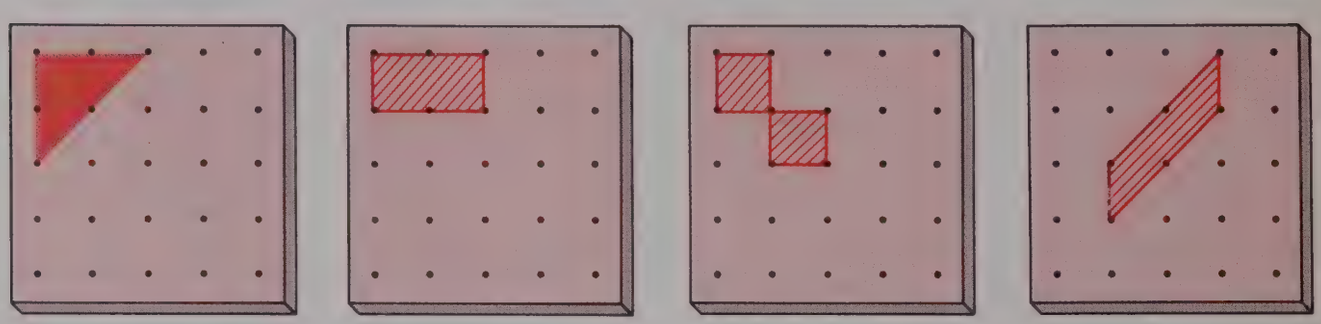
1. $\frac{1}{2}$



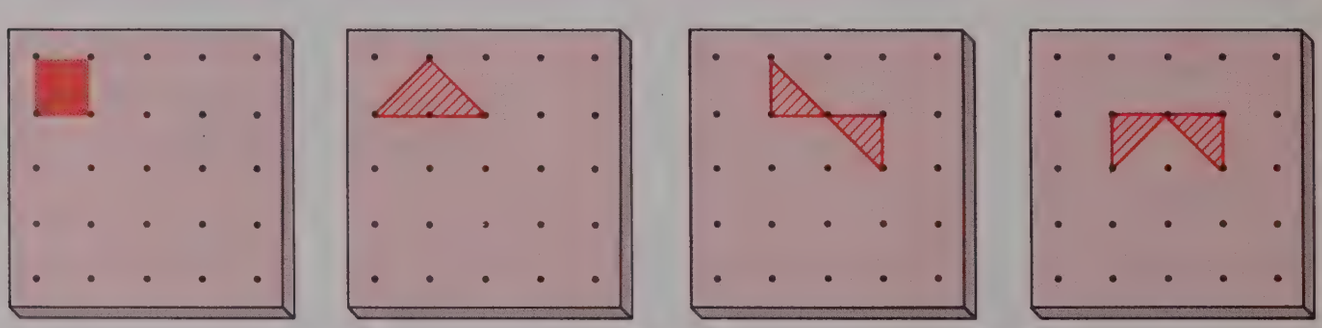
2. $\frac{1}{4}$



3. $\frac{1}{8}$



4. $\frac{1}{16}$



5. The "area" of each nailboard is 16. Can you use the multiplication equations to give the area of the regions you found above?

A $\frac{1}{2} \times 16 = n$ $n = 8$

B $\frac{1}{4} \times 16 = n$ $n = 4$

C $\frac{1}{8} \times 16 = n$ $n = 2$

D $\frac{1}{16} \times 16 = n$ $n = 1$

Multiplying with Whole Numbers and Unit Fractions

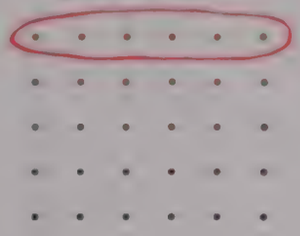
1. Ring the part of the set of dots indicated. Then write a multiplication equation suggested by the part of the set.

A $\frac{1}{4}$ of 12

Equation

$\frac{1}{4} \times 12 = 3$

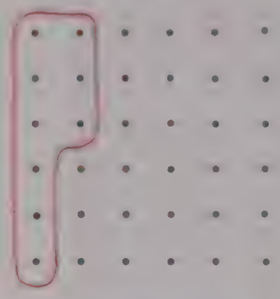
B $\frac{1}{5}$ of 30



Equation

$\frac{1}{5} \times 30 = 6$

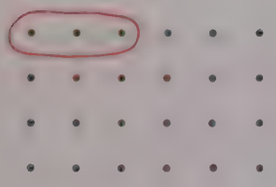
C $\frac{1}{4}$ of 36



Equation

$\frac{1}{4} \times 36 = 9$

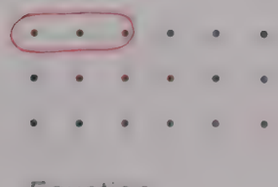
D $\frac{1}{8}$ of 24



Equation

$\frac{1}{8} \times 24 = 3$

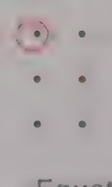
E $\frac{1}{6}$ of 18



Equation

$\frac{1}{6} \times 18 = 3$

F $\frac{1}{9}$ of 9



Equation

$\frac{1}{9} \times 9 = 1$

2. Find the products.

A $\frac{1}{5} \times 20 = 4$

B $\frac{1}{10} \times 100 = 10$

C $\frac{1}{6} \times 48 = 8$

D $\frac{1}{8} \times 56 = 7$

E $\frac{1}{7} \times 63 = 9$

F $\frac{1}{3} \times 27 = 9$

G $\frac{1}{9} \times 45 = 5$

H $\frac{1}{4} \times 84 = 21$

I $\frac{1}{13} \times 91 = 7$

3. Complete each sentence by giving the missing number or numbers.

A $20 \div 2 = 20 \times \frac{1}{2} = 10$

B $15 \div 3 = 15 \times \frac{1}{3} = 5$

C $24 \div 6 = 24 \times \frac{1}{6} = 4$

D $42 \div 7 = 42 \times \frac{1}{7} = 6$

E $36 \div 4 = 36 \times \frac{1}{4} = 9$

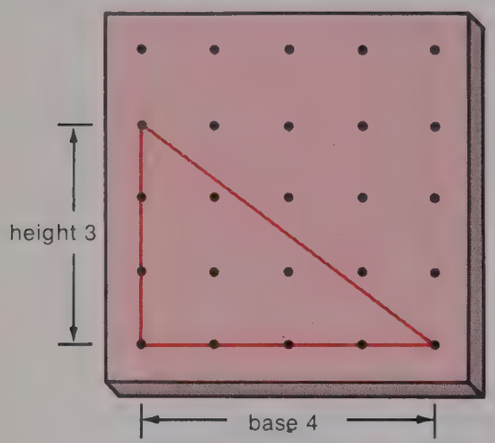
F $10 \div 2 = 10 \times \frac{1}{2} = 5$

G $65 \div 13 = 65 \times \frac{1}{13} = 5$

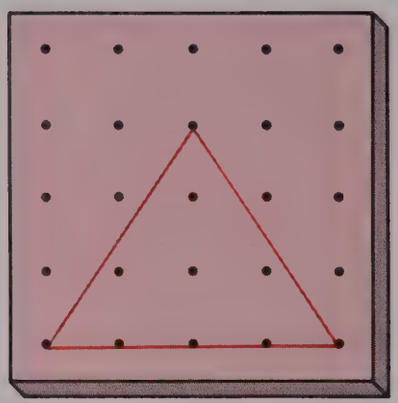
H $120 \div 8 = 120 \times \frac{1}{8} = 15$

Triangle Area on the Geoboard

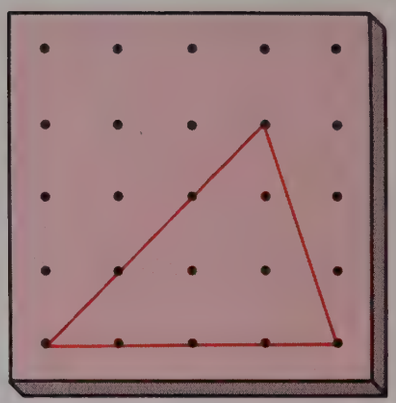
1. How many other triangles with different shapes can you find that have base 4 and height 3? Give the area of each one you find.



Area 6

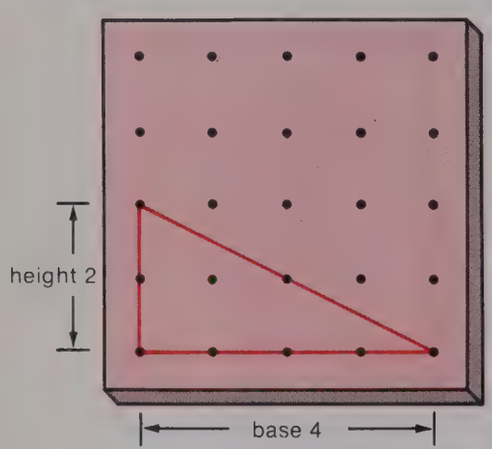


Area 6

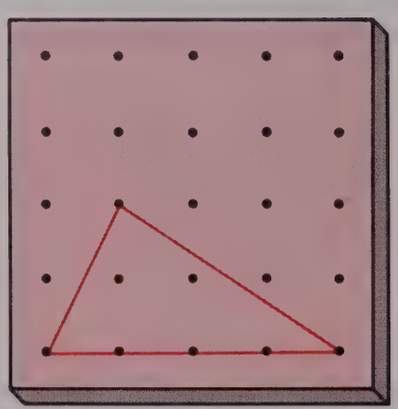


Area 6

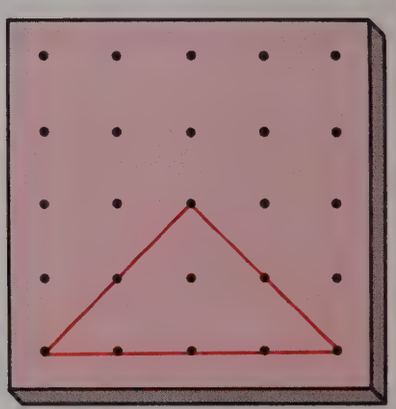
2. How many other triangles with different shapes can you draw that have base 4 and height 2? Find the area of each.



Area 4

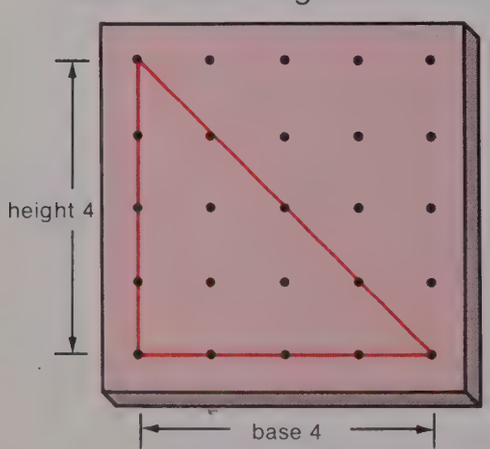


Area 4

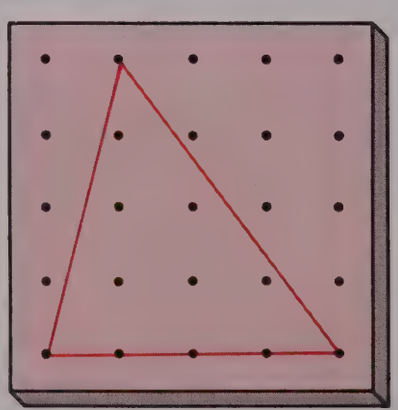


Area 4

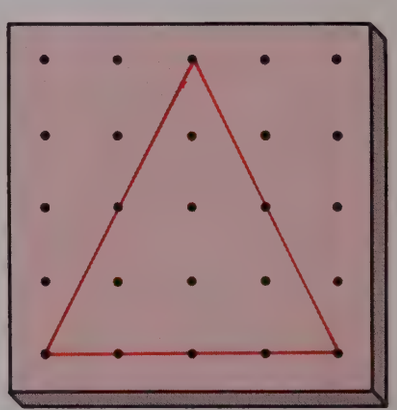
3. How many other triangles with different shapes can you draw that have base 4 and height 4? Give the area of each.



Area 8



Area 8



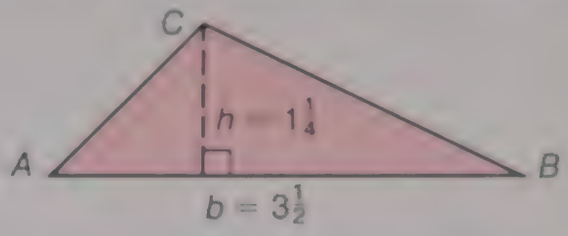
Area 8

Multiplying Fractional Numbers -- Area of Triangles

The area A of any triangular region is the product of $\frac{1}{2}$ the **base** b times the **height** h.

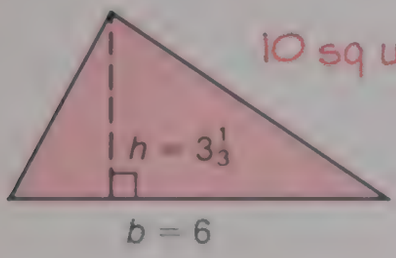
For $\triangle ABC$.

$$A = \tfrac{1}{2} \times b \cdot h$$
$$= \tfrac{1}{2} \times 3\tfrac{1}{2} \times 1\tfrac{1}{4}$$
$$= \tfrac{1}{2} \times \tfrac{7}{2} \times \tfrac{5}{4} = \tfrac{35}{16} \text{ or } 2\tfrac{3}{16} \text{ sq units}$$



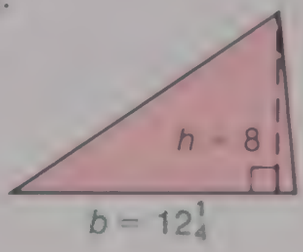
Find the area of these triangular regions in Exercise 1 through 6.

1.



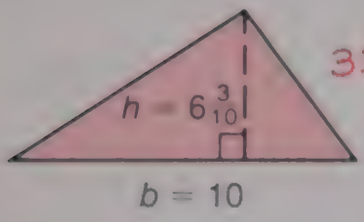
10 sq units

2.



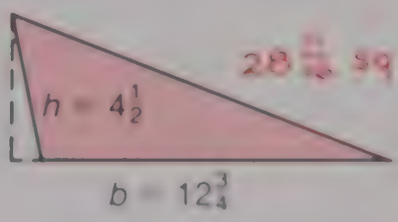
49 sq units

3.



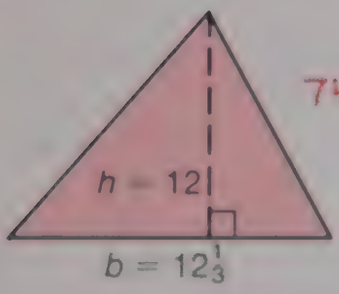
33 sq units

4.



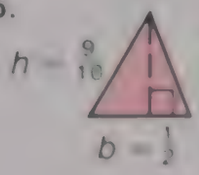
$28\frac{5}{8}$ sq units

5.



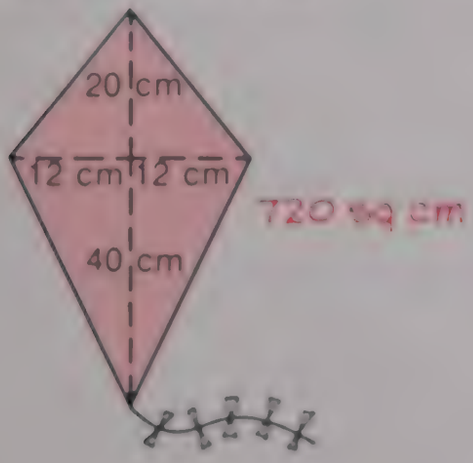
74 sq units

6.



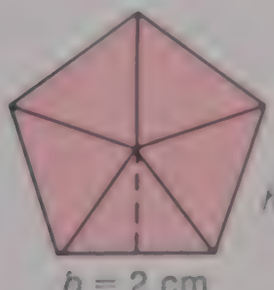
$\frac{9}{40}$ sq units

7. Find the area of this kite.



720 sq cm

8. Find the area of the pentagonal region. All the triangles are the same size and shape.



1 sq cm

Fractional Number Factor Trees

Give the missing fractional number for each.

1. $\frac{1}{2} \times \boxed{\frac{3}{2}} \times \frac{2}{5} \times \frac{1}{3}$

$\frac{3}{4} \times \boxed{\frac{2}{15}}$

$\boxed{\frac{6}{60}}$

2. $\frac{1}{5} \times \boxed{\frac{1}{2}} \times \frac{3}{5} \times \boxed{\frac{1}{2}}$

$\frac{1}{10} \times \frac{3}{10}$

$\boxed{\frac{3}{100}}$

3. $3 \times \boxed{\frac{1}{2}} \times \frac{5}{4} \times \boxed{\frac{1}{2}}$

$\frac{3}{2} \times \boxed{\frac{5}{8}}$

$\frac{15}{16}$

4. $4 \times \boxed{\frac{1}{5}} \times 3 \times \boxed{\frac{1}{7}}$

$\frac{4}{5} \times \frac{3}{7}$

$\boxed{\frac{12}{35}}$

5. $\frac{1}{5} \times \boxed{\frac{1}{4}} \times \boxed{\frac{1}{3}} \times \frac{1}{2}$

$\frac{1}{20} \times \boxed{\frac{1}{6}}$

$\frac{1}{120}$

6. $2 \times 3 \times \frac{1}{5} \times \frac{1}{5}$

$\boxed{6} \times \boxed{\frac{1}{25}}$

$\boxed{\frac{6}{25}}$

7. $\boxed{8} \times 3 \times \boxed{\frac{1}{5}} \times \boxed{\frac{1}{7}}$

$24 \times \boxed{\frac{1}{35}}$

$\frac{24}{35}$

8. $\boxed{7} \times \boxed{7} \times \boxed{\frac{1}{10}} \times \frac{1}{10}$

$\boxed{49} \times \frac{1}{100}$

$\frac{49}{100}$

9. Make your own "factor tree" for $\frac{25}{64}$.

$5 \times 5 \times \frac{1}{8} \times \frac{1}{8}$

$25 \times \frac{1}{64}$

$\frac{25}{64}$

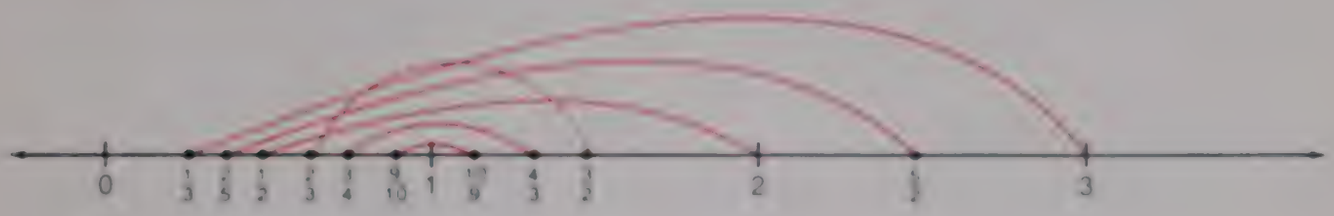
$5 \times 5 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$5 \times 5 \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2}$

In exercise 9 the top row of the factor tree will be the same only if prime whole numbers or unit fractions with prime denominators are used.

Reciprocals

1. Connect the point for each fractional number shown (except 0) with its reciprocal.



2. Complete each sentence below. Use the number line above to help you.

   A The reciprocal of $\frac{99}{100}$ is $\frac{100}{99}$.

   B The reciprocal of $\frac{999}{1000}$ is $\frac{1000}{999}$.

   C Numbers which are very close to 1 have reciprocals which are very close to 1.

   D The reciprocal of 10 is $\frac{1}{10}$.

   E The reciprocal of 1000 is $\frac{1}{1000}$.

   F The larger the number the nearer its reciprocal will be to 0.

   G The number 1 has its own reciprocal.

   H The number 0 has no reciprocal.

   I If n is any whole number except 0, the reciprocal of n is $\frac{1}{n}$.

3. A Find the reciprocals of 2, 3, and 6.

   $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$

   B Find the sum of the reciprocals. 1

4. A Give the reciprocals of 2, 4, 7, 14, and 28.

   $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{28}$

   B Find the sum of the reciprocals 1

Division Short Story Problems

Write and solve a division equation for each short story problem.

1. Traveled 247 km in $3\frac{1}{4}$ hrs. How many km/h?

Solution:
$247 \div 3\frac{1}{4} = n$

$n =$ 76

2. Jet airplane traveled 3200 km in $5\frac{1}{3}$ hours. How many km/h? 600 km/h

3. Walked $\frac{3}{5}$ km in 9 minutes. How long to walk 1 km? 15 min

4. Slow snail moved 14 cm in $3\frac{1}{2}$ minutes. How many cm/min? 4 cm/min

5. Ran 100 meters in 11 seconds. How many m/sec? $9\frac{1}{11}$ m/sec

6. Speedy spider "ran" 3 cm in 1 second. How many seconds to run a meter? $33\frac{1}{3}$ sec

7. Freight train goes 180 kilometers in $3\frac{3}{4}$ hours. What is the train's speed? 48 km/h

8. A cheetah can sprint about 90 kilometers per hour. How many meters per second is this? 25 m/sec

9. A pike fish can swim 16 km in one hour. How far could a pike swim in 15 minutes? 4 km

10. Distance around the earth: 40,000 km.
Satellite speed 24,000 km/h.
How long for satellite to travel around the earth? $1\frac{2}{3}$ hr or 1 hr 40 min

Continued Fractions

1. The reciprocal of $\frac{2}{3}$ can be written as $\frac{1}{\frac{2}{3}} = 1 \div \frac{2}{3} = 1 \times \frac{3}{2} = \frac{3}{2}$.

   Simplify each of the following fractions.

   A $\frac{1}{\frac{5}{4}} = \frac{4}{5}$ B $\frac{1}{\frac{1}{2}} = 2$ C $\frac{1}{\frac{3}{5}} = \frac{5}{3}$

   D $\frac{1}{\frac{1}{8}} = 8$ E $\frac{1}{\frac{3}{7}} = \frac{7}{3}$ F $\frac{1}{\frac{7}{10}} \quad \frac{10}{7}$

2. Express each fraction as a simple fraction of the form $\frac{a}{b}$.

   A $\frac{1}{1+\frac{1}{3}} \quad \frac{1}{\frac{4}{3}} = \frac{3}{4}$ B $\frac{1}{2+\frac{1}{2}} \quad \frac{2}{5}$ C $\frac{1}{3+\frac{1}{5}} \quad \frac{5}{16}$

   D $\frac{1}{1+\frac{1}{2}} \quad \frac{2}{3}$ E $\frac{1}{2+\frac{1}{4}} \quad \frac{4}{9}$ F $\frac{1}{1+\frac{1}{7}} \quad \frac{7}{8}$

3. An expression such as $1 + \cfrac{1}{1 + \cfrac{1}{1+\frac{1}{2}}}$ is called a **continued fraction**.

   To simplify such an expression you begin at the lower right corner of the expression and "work your way back up."

   $1 + \cfrac{1}{1 + \cfrac{1}{1+\frac{1}{2}}} = 1 + \cfrac{1}{1 + \cfrac{1}{\frac{3}{2}}} = 1 + \cfrac{1}{\frac{5}{3}} = 1 + \frac{3}{5} = \frac{8}{5}$

   Simplify these continued fractions.

   A $1 + \cfrac{1}{1+\frac{1}{3}} \quad \frac{7}{4}$ B $1 + \cfrac{1}{2+\frac{1}{2}} \quad \frac{7}{5}$

   C $2 + \cfrac{1}{3+\frac{1}{4}} \quad \frac{30}{13}$ D $1 + \cfrac{1}{1 + \cfrac{1}{1+\frac{1}{5}}} \quad \frac{17}{11}$

4. Can you find a continued fraction for $\frac{9}{5}$?

   $1 + \cfrac{1}{1+\frac{1}{4}}$

Speeds

1. Walked $7\frac{1}{2}$ km in $1\frac{2}{3}$ hours. How many km per hour?

$4\frac{1}{2}$ km/h

2. Bicyclist: 56 km in $5\frac{1}{3}$ hours. How fast? $10\frac{1}{2}$ km/h

3. Snail's pace: 7 cm in $2\frac{1}{4}$ minutes. How many cm per minute?

$3\frac{1}{9}$ cm/min

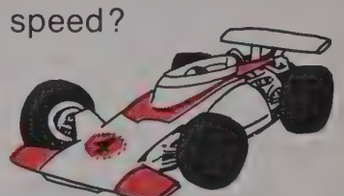
4. Cheetah: 100 meters in $4\frac{2}{5}$ seconds. How many meters per second?

$22\frac{8}{11}$ m/sec

5. Auto speed limit: 90 km/h. How long to travel 225 km at this speed? $2\frac{1}{2}$ hrs

6. 800 km auto race. Winner finishes in $3\frac{1}{5}$ hours. What is the winner's speed?

250 km/h



7. Pronghorn antelope: Runs 200 meters in $9\frac{1}{10}$ seconds. How long to run a kilometer?

$45\frac{1}{2}$ seconds

8. Olympic runner: 100 meters in $9\frac{9}{10}$ seconds. How many meters per second?

$10\frac{10}{99}$ m/sec

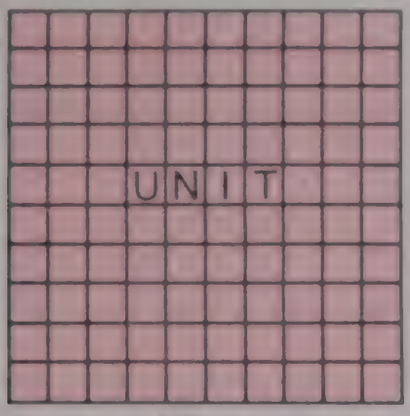
9. Horse race: 2 km. Time: $2\frac{2}{15}$ minutes. How many km per hour?

$56\frac{1}{4}$ km/h

10. Persian greyhound runs 63 km/h. How many meters per second?

$17\frac{1}{2}$ m/sec

Use the large square as the unit. The areas of the other 2 regions are given as decimal parts of the unit square.



$1 = \frac{100}{100} = 1.00$

$\frac{1}{10} = \frac{10}{100} = 0.1 = 0.10$

$\frac{1}{100} = 0.01$

Write a decimal that shows the area of each region as a decimal.

1. 

.18

2. 

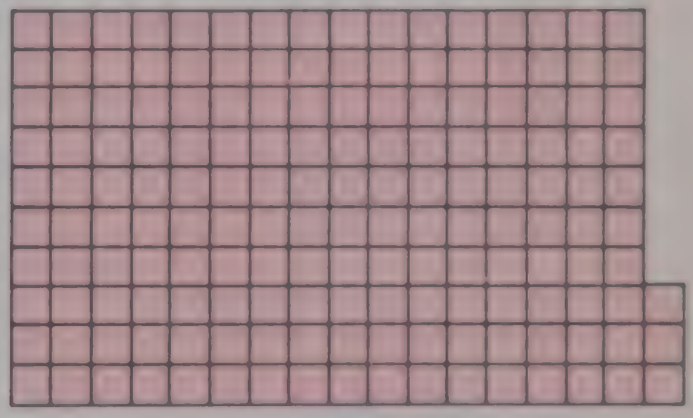
.25

3. 

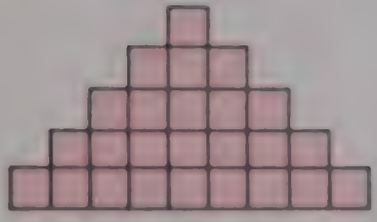
.13

4. 

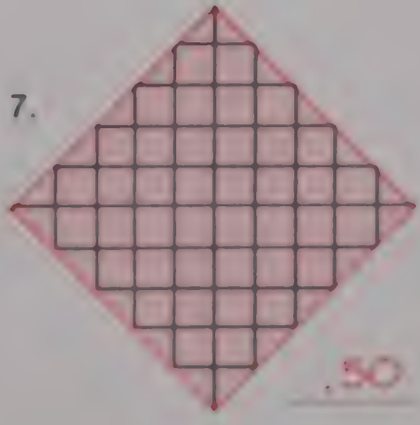
.37

5. 

1.63

6. 

.25

7. 

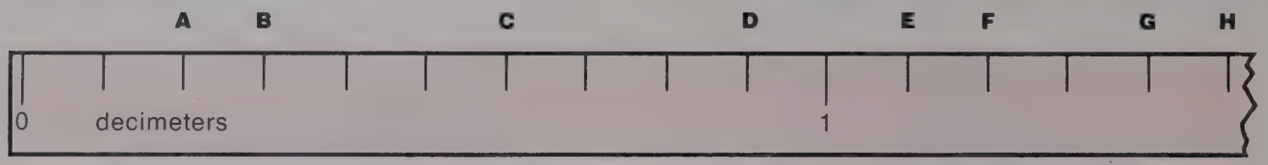
.50

8. 

.46

Metric System and Decimals

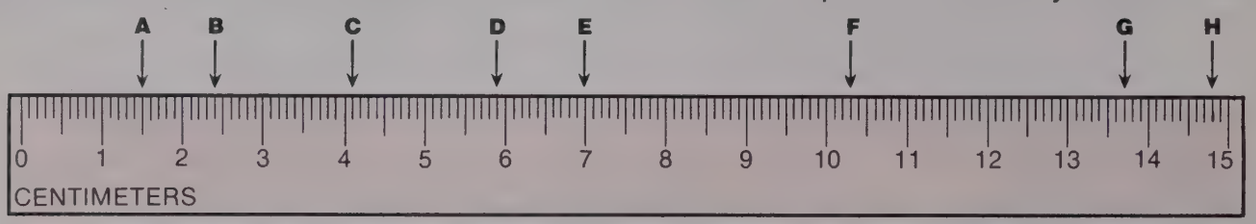
1. Give a decimal for the length in decimeters from 0 to the given point on the ruler.



A .20 dm B .30 dm C .60 dm D .90 dm

E 1.1 dm F 1.2 dm G 1.4 dm H 1.5 dm

2. Give a decimal for the measure in centimeters for the point shown by the arrow.



A 1.5 cm B 2.4 cm C 4.1 cm D 5.9 cm

E 7.0 cm F 10.3 cm G 13.7 cm H 14.8 cm

3. Express each measure of Exercise 2 in decimeters.

A .15 dm B .24 dm C .41 dm D .59 dm

E .70 dm F 1.03 dm G 1.37 dm H 1.48 dm

4. Give each measure as a decimal in meters. Use the table of relations.

1 dm = 0.1 meters
1 cm = 0.01 meters
1 mm = 0.001 meters

A 3 m 4 dm 5 cm 7 mm = 3.457 m

B 7 m 1 dm 3 cm 2 mm = 7.132 m

C 8 dm 5 cm 4 mm = .854 m

D 16 m 8 dm = 16.8 m

E 1 m 2 dm 9 cm = 1.29 m

F 23 m 6 cm = 23.06 m

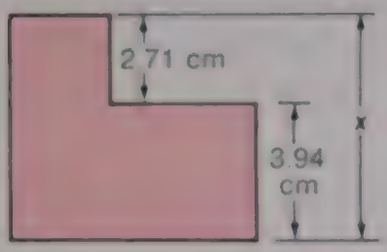
G 2 dm 8 mm = .208 m

H 6 cm 1 mm = .061 m

Adding and Subtracting with Decimals

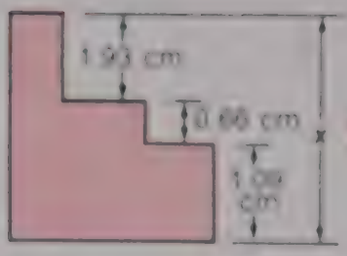
Find length **x** in each drawing.

1.



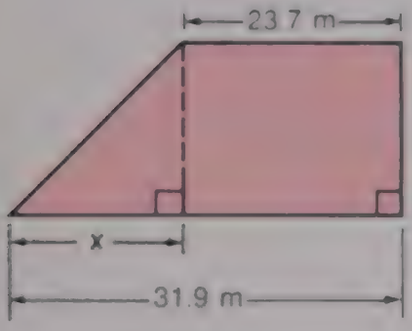
x = 6.65 cm

2.



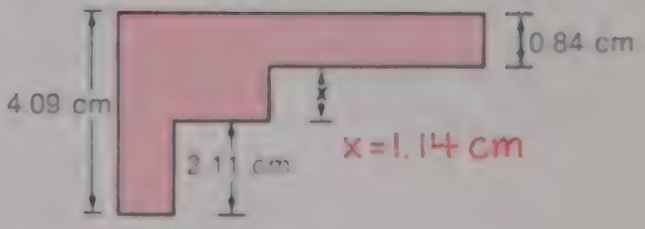
x = 3.68 cm

3.



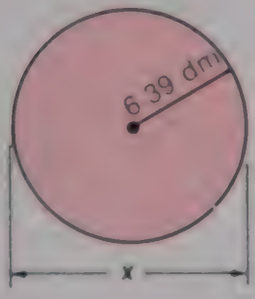
x = 8.2 m

4.



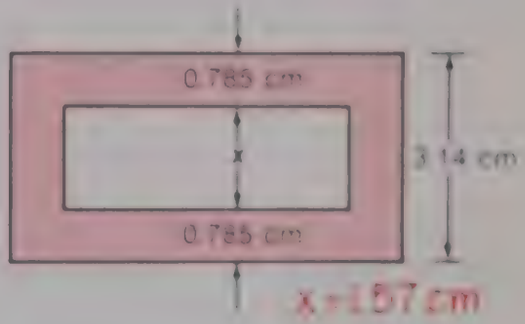
x = 1.14 cm

5.



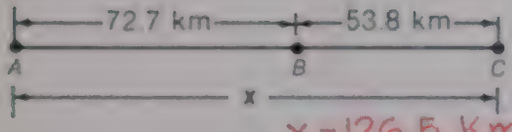
x = 12.78

6.



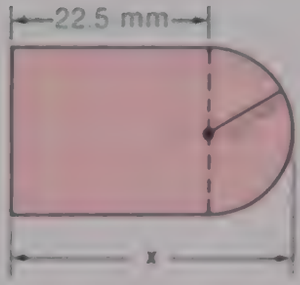
x = 1.57 cm

7.



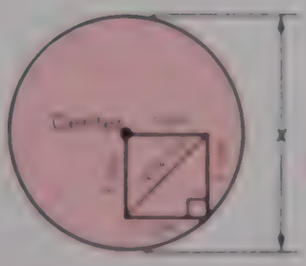
x = 126.5 Km

8.



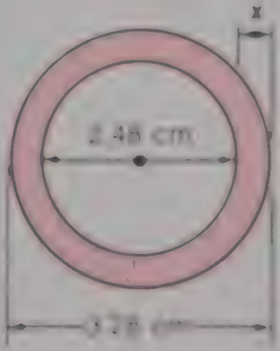
x = 31.8 mm

9.



x = 2.828 cm

10.



x = .40 cm

Rounding Decimals

1. Round the decimal in each sentence to the nearest hundredth.

A A straight pin has a mass of about .126 grams.

.13

B Sound travels about .332 km/sec in air.

.33

C Sound travels about 1.404 km/sec in water.

1.40

D A penny has a mass of about 3.118 grams.

3.12

2. Round each decimal to the nearest thousandth.

A The circumference of a circle is about 3.14159 times the diameter of the circle.

3.142

B A cubic decimeter of air has a mass of .00129 kg.

.001

C The volume of a sphere is about 1.0472 times the square of the diameter of the sphere.

1.047

D The height of an equilateral triangle one centimeter on each side is about .86602 cm.

.866

3. Round each number to the nearest tenth. Write your answer in the corresponding lettered square. If all of your answers are correct, the numbers should form a magic square.

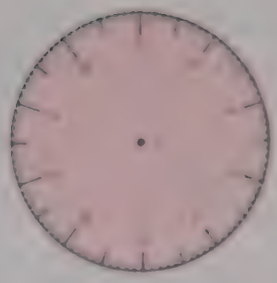
A 1.6	B .2	C .3	D 1.3
E .5	F 1.1	G 1.0	H .8
I .9	J .7	K .6	L 1.2
M .4	N 1.4	O 1.5	P .1

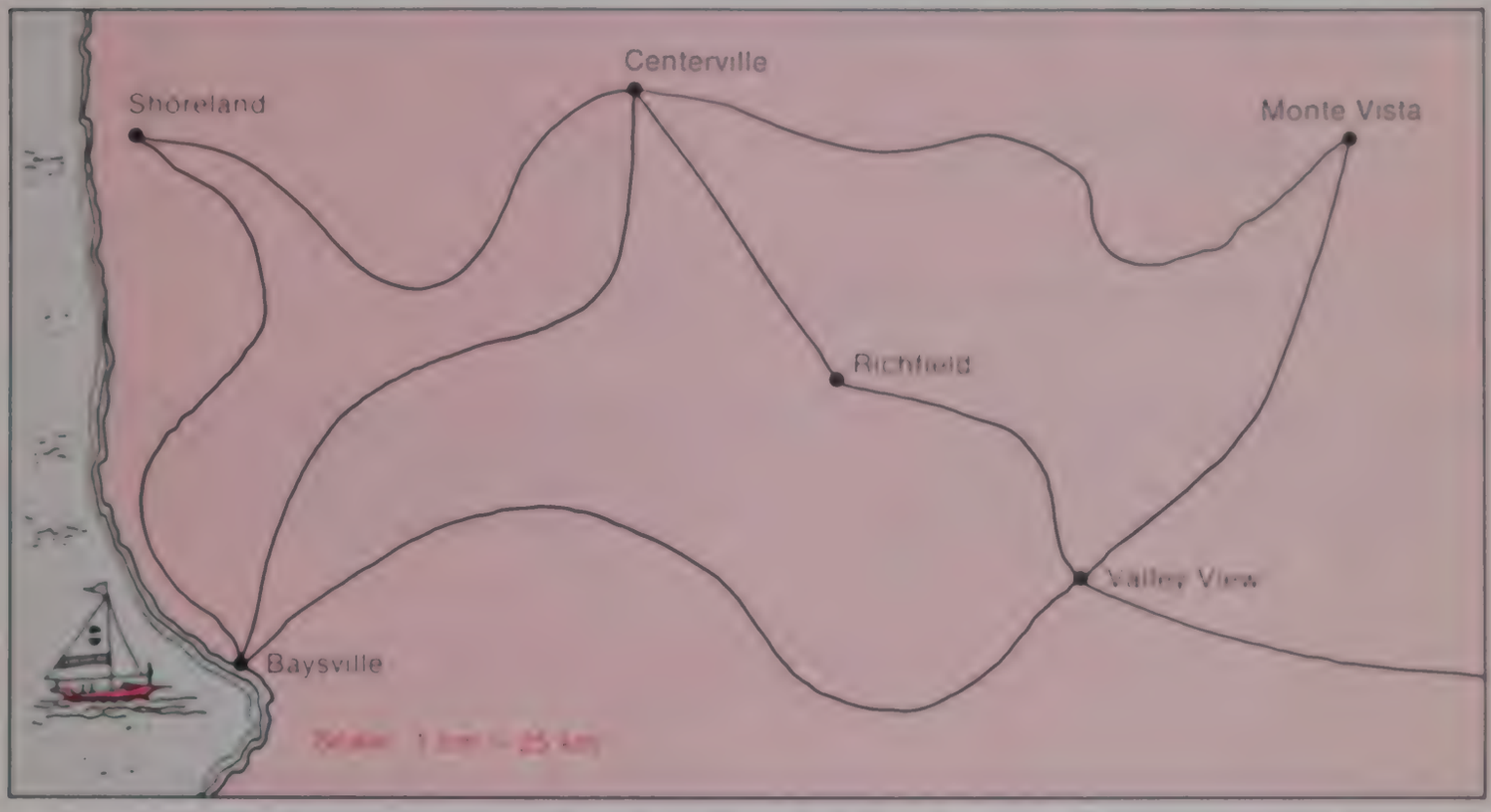
A 1.5761	**B** 0.247	**C** 0.25761	**D** 1.30999
E 0.47631	**F** 1.09	**G** 0.9628	**H** 0.83333
I 0.86666	**J** 0.6822	**K** 0.61	**L** 1.1609
M 0.4428	**N** 1.3828	**O** 1.5271669	**P** 0.082

4. What is the "magic sum" for the square of Exercise 3? 3.4

Map Distances

Trace and cut out the centimeter trundle wheel. Paste it on a piece of cardboard. Push your pencil point through the center of the wheel. Roll your trundle wheel along the roads of the map. Measure each distance to the nearest tenth of a centimeter. Complete the table below.





Cities	Map Distance (cm)	Actual Distance (km)
Shoreland to Baysville	7.9	197.5
Centerville to Richfield	4.3	107.5
Monte Vista to Centerville	10.0	250.0
Baysville to Valley View	12.2	305.0
Valley View to Monte Vista	6.5	162.5
Richfield to Valley View	4.2	105.0
Baysville to Centerville	9.4	235.0

Measurements will vary somewhat due to inaccuracies in measuring along the curved roads.

Dividing with Decimals

1. Study the whole-number division problem in the box. Then give the quotient for each of the division problems below. Try to avoid dividing.

A $2.9\overline{)182.12}$ 62.8

B $2.9\overline{)1.8212}$ .628

C $0.29\overline{)1.8212}$ 6.28

D $0.29\overline{)182.12}$ 628

E $0.29\overline{)1821.2}$ 6280

F $0.029\overline{).18212}$ 6.28

G $29\overline{)18.212}$ .628

H $0.029\overline{)1821.2}$ 62800

$$\begin{array}{r} 628 \\ 29\overline{)18{,}212} \\ 17\,4 \\ \hline 81 \\ 58 \\ \hline 232 \\ 232 \\ \hline 0 \end{array}$$

Solve each problem in Exercises 2 through 7.

2. One of the hairs in your head is about 0.008 cm thick. About how many hairs, laid side by side, would make a width of 1 cm? 125

3. The distance around the outside of a bicycle tire is 2.07 meters. How many turns will the bicycle wheel make in traveling 1 km? 483.09

4. An automobile used 33.5 liters of gasoline in traveling 308.2 km. How many kilometers per liter did the automobile get? 9.2 km/ℓ

5. Suppose an ant has a "pace" of .16 cm. How many paces must the ant take in crossing a room which is 4.8 meters wide? 3000

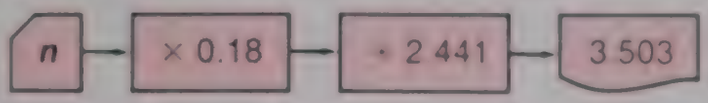
6. A sheet of paper is about .0075 cm thick. How many pages in a tablet 8 mm thick? about 107

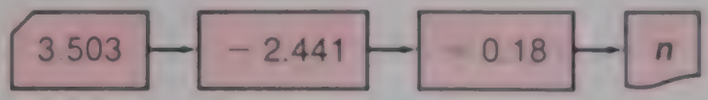
7. If you were on the moon your weight would be only .16 of your weight on earth. What would be the earth weight of a person whose moon weight is 13.12 kg? 82 kg

Flow Charts and Equations

1. Study the equation, its flow chart, and the inverse flow chart. Then find the output number *n*. This is the solution for the equation.

Equation: $(n \cdot 0.18) + 2.441 = 3.503$

Flow Chart: 

Inverse Flow Chart: 

$n =$ 5.9

2. Complete the inverse flow chart and find the number for *n* in the equation.

Equation: $(n \times 6.2) - 8.49 = 73.97$

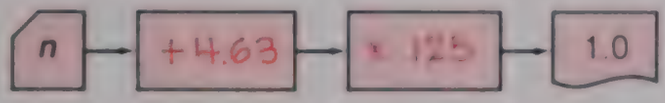
Flow Chart: 

Inverse Flow Chart: 

$n =$ 13.3

3. Complete the flow charts and find the number for *n*.

Equation: $(n + 4.63) \times .125 = 1.0$

Flow Chart: 

Inverse Flow Chart: 

$n =$ 3.37

4. Complete each flow chart. Then give the number for *n*

Equation: $(17.22 \times n) \div 2.1 = 67.24$

Flow Chart: 

Inverse Flow Chart: 

$n =$ 8.2

5. Solve these equations. Make flow charts if necessary.

▲ $(n - 2.42) \times 4.1 = 26.814$
$n = 8.96$

■ $(n \cdot 0.1) \cdot 0.88 = 0.999$
$n = 1.19$

Help the children understand that in the inverse flow chart the inverse operations are used in a reversed order from the operations of the original flow chart.

Repeating Decimals

1. Repeating decimals are often written using a bar to denote the repeating part of the decimal.

EXAMPLE 1: $0.6666\ldots = .6$

EXAMPLE 2: $0.723723\ldots = .\overline{723}$

EXAMPLE 3: $26.492929\ldots = 26.\overline{492}$

Use bar notation to represent each repeating decimal.

A $0.736736\ldots =$ $.\overline{736}$

B $2.677777\ldots =$ $2.6\overline{7}$

C $0.12341234\ldots =$ $.\overline{1234}$

D $3.090909\ldots =$ $3.\overline{09}$

E $0.999999\ldots =$ $.\overline{9}$

F $0.00414141\ldots =$ $.00\overline{41}$

2. Find the sums. Add from left to right.

A
$$\begin{array}{r} .2\,9\,2\,9\,2\,9\ldots \\ +\,.3\,4\,3\,4\,3\,4\ldots \\ \hline .6\,3\,6\,3\,6\,3\ldots \end{array}$$

B
$$\begin{array}{r} .1\,7\,6\,1\,7\,6\,1\,7\,6\ldots \\ +\,.4\,4\,4\,4\,4\,4\,4\,4\,4\ldots \\ \hline .6\,2\,0\,6\,2\,0\,6\,2\,0\ldots \end{array}$$

C
$$\begin{array}{r} .5\,7\,6\,5\,7\,6\,5\,7\,6\ldots \\ +\,.2\,3\,2\,3\,2\,3\,2\,3\,2\ldots \\ \hline .8\,0\,8\,8\,9\,9\,8\,0\,8\ldots \end{array}$$

D
$$\begin{array}{r} .6\,6\,6\,6\,6\,6\,6\ldots \\ +\,.4\,4\,4\,4\,4\,4\,4\ldots \\ \hline 1.1\,1\,1\,1\,1\,1\,1\ldots \end{array}$$

3. Find the difference. Work from left to right.

A
$$\begin{array}{r} .6\,7\,8\,6\,7\,8\,6\,7\,8\ldots \\ -\,.1\,3\,1\,3\,1\,3\,1\,3\,1\ldots \\ \hline .5\,4\,7\,3\,6\,5\,5\,4\,7\ldots \end{array}$$

B
$$\begin{array}{r} .9\,2\,9\,2\,9\,2\,9\,2\ldots \\ -\,.6\,4\,6\,4\,6\,4\,6\,4\ldots \\ \hline .2\,8\,2\,8\,2\,8\,2\,8\ldots \end{array}$$

C
$$\begin{array}{r} .1\,0\,9\,1\,0\,9\,1\,0\,9\ldots \\ -\,.0\,8\,8\,0\,8\,8\,0\,8\,8\ldots \\ \hline .0\,2\,1\,0\,2\,1\,0\,2\,1\ldots \end{array}$$

D
$$\begin{array}{r} 1.0\,0\,0\,0\,0\,0\,0\,0\ldots \\ -\,.9\,9\,9\,9\,9\,9\,9\,9\ldots \\ \hline .0\,0\,0\,0\,0\,0\,0\,0\ldots \end{array}$$

4. Multiply. Work from left to right.

$$\begin{array}{r} .3\,3\,3\,3\,3\ldots \\ \times\ 4 \\ \hline 1.3\,3\,3\,3\,3\ldots \end{array}$$

$$\begin{array}{r} .1\,2\,1\,2\,1\,2\,1\,2 \\ \times\ 5 \\ \hline .6\,0\,6\,0\,6\,0\,6\,0\ldots \end{array}$$

C
$$\begin{array}{r} .9\,9\,9\,9\,9\,9\ldots \\ \times\ 9 \\ \hline 8.9\,9\,9\,9\,9\ldots \end{array}$$

D
$$\begin{array}{r} .2\,6\,3\,2\,6\,3\,2\,6\,3 \\ \times\ 6 \\ \hline 1.5\,7\,9\,5\,7\,9\,5\,7\ldots \end{array}$$

In working "from left to right," children must think about the possible regrouping necessary from the next decimal place on the right.

Scientific Notation

1. Complete the powers of 10 table below.

Positive Exponents	Zero Exponent	Negative Exponents
$10^1 = 10$		$10^{-1} = \frac{1}{10} = .1$
$10^2 = 100$		$10^{-2} = \frac{1}{10^2} = .01$
$10^3 = 1000$	$10^0 = 1$	$10^{-3} = \frac{1}{10^3} = .001$
$10^4 = 10,000$		$10^{-4} = \frac{1}{10^4} = .0001$
$10^5 = 100000$		$10^{-5} = \frac{1}{10^5} = .00001$
.		.
.		.
.		.

2. Numbers which are greater than or equal to 1 can be expressed in scientific notation using powers of 10 with zero or positive number exponents. Express each number in scientific notation by giving the missing power of ten.

A $350,000 = 3.5 \times 10^5$

B $7280 = 7.28 \times 10^3$

C $6,100,000 = 6.1 \times 10^6$

D $37 = 3.7 \times 10^1$

E $8.2 = 8.2 \times 10^0$

F $1000 = 1.0 \times 10^3$

3. Decimals for numbers between 0 and 1 can be expressed in scientific notation using negative exponents for the powers of ten.

EXAMPLE 1: $0.0036 = 3.6 \times \frac{1}{10^3} = 3.6 \times 10^{-3}$

EXAMPLE 2: $0.0000793 = 7.93 \cdot 10^{-5}$

Express each number in scientific notation.

A $0.025 = 2.5 \times 10^{-2}$

B $0.00077 = 7.7 \times 10^{-4}$

C $0.00004 = 4.0 \times 10^{-5}$

D $0.000000828 = 8.28 \times 10^{-7}$

E $0.027 = 2.7 \times 10^{-2}$

F $0.162 = 1.62 \times 10^{-1}$

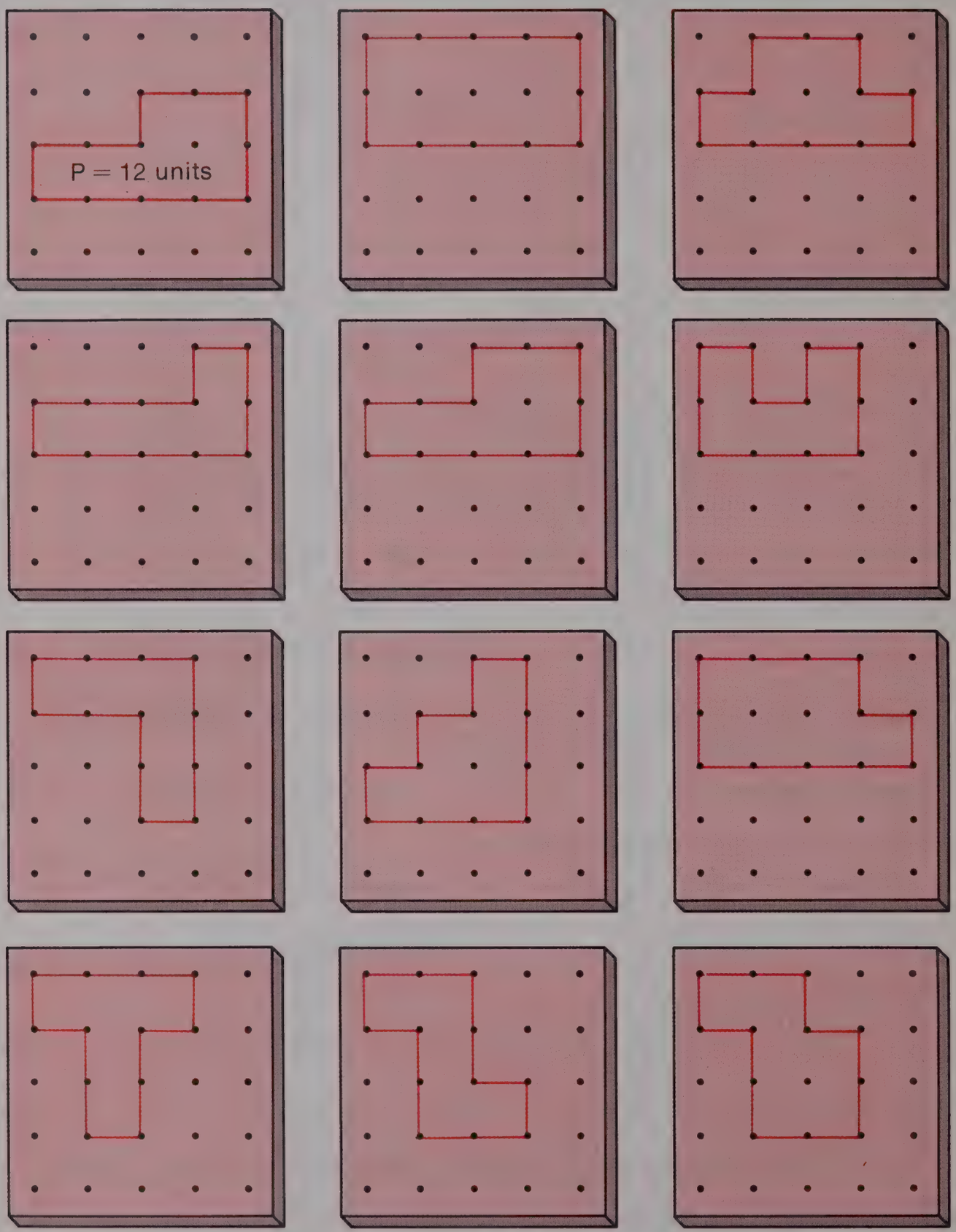
4. The mass of 1 cubic decimeter of uranium is about 18,700 grams. Express the mass in scientific notation. 1.87×10^4

5. The mass of one cubic decimeter of hydrogen gas is about .09 grams. Express the mass in scientific notation. 9.0×10^{-2}

Encourage children to use the table in exercise 1 as an aid in working the remaining exercises.

The first geoboard shows a polygon with a perimeter of 12 units. Show another polygon of different shape but with a perimeter of 12 units on each of the other geoboards.

Some sample solutions are shown



Children can carry out the activity using actual geoboards, then record their polygons on the page.

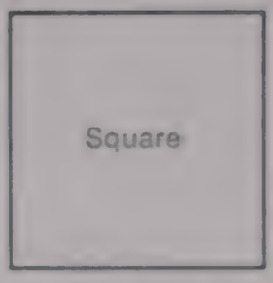
Perimeters of Regular Polygons

A **regular polygon** has all sides the same length and all angles congruent.
Measure **one** edge of each regular polygon to the nearest **tenth** of a centimeter.
Then compute the perimeter.

1. 

s = 1.7 cm

Perimeter = 6 × s = 10.2 cm

2. 

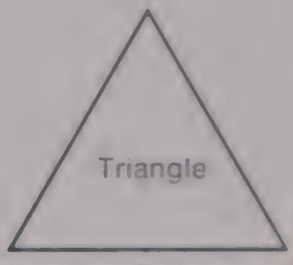
s = 3.1 cm

Perimeter = 4 × s = 12.4 cm

3. 

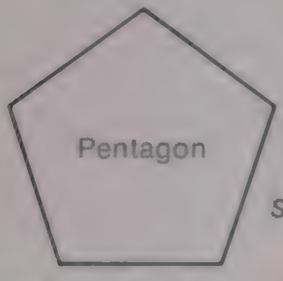
s = 1.3 cm

Perimeter = 8 × s = 10.4 cm

4. 

s = 3.4 cm

Perimeter = 3 × s = 10.2 cm

5. 

s = 2.0 cm

Perimeter = 6 × s = 12.0 cm

6. 

s = 1.4 cm

Perimeter = 7 × s = 9.8 cm

7. 

s = 1.1 cm

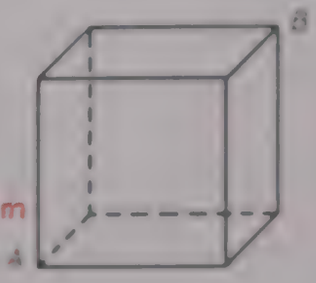
Perimeter = 10 × s = 11 cm

8. 

s = .9 cm

Perimeter = 12 × s = 10.8 cm

9. Each edge of the cube is 3.7 cm long.
   - A What is the length of the shortest path **along the edge** from A to B? 11.1 cm
   - B What is the longest path from A to B **along the edge** if no point can be touched twice? 25.9 cm



Finding π

In each figure, the length of $\overline{AB}$ represents the circumference C of the circle. Measure C and diameter d to the nearest tenth of a centimeter, then compute $C \div d$ to the nearest tenth.

1.

$C \div d =$ 3.1

$d =$ 3.5

$C =$ 10.9 cm

A B

2.

$C \div d =$ 3.1

$d =$ 3.0

$C =$ 9.4

A B

3.

$C = d =$ 3.1

$d =$ 3.9

$C =$ 12.2

A B

4.

$C \div d =$ 3.2

$d =$ 2

$C =$ 6.3

A B

5.

$C \div d =$ 3.1

$d =$ 2.7

$C =$ 8.5

A B

6. What is the average of the five numbers you found for $C \div d$ in Exercises 1 to 5? 3.12 answers may vary

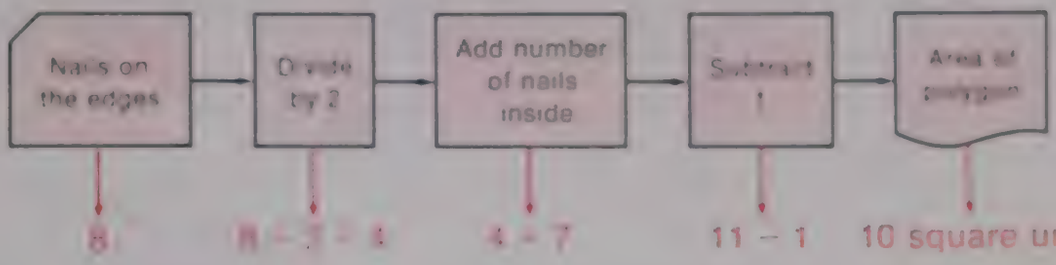
The number should be near to the number $\pi = 3.14159\ldots$.

How close are you? .02 answers may vary

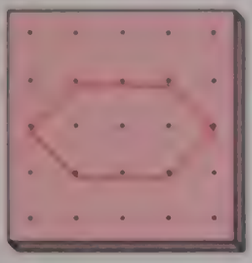
As a follow up activity, have children cut a paper strip as long as AB and tape the ends together to form a circle. See if the paper circle matches the circle on the page.

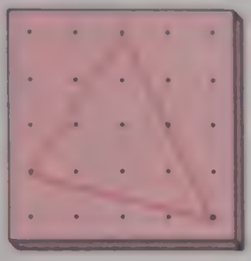
Pick's Area Formula

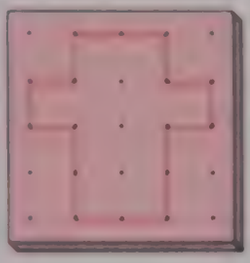
The flow chart below gives instructions for finding the area of any simple polygon on the geoboard. The formula for the area is often called **Pick's Formula**

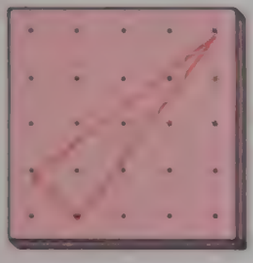


Find the area of each polygon using Pick's Formula in exercises 1–9.

1. 
6 sq units

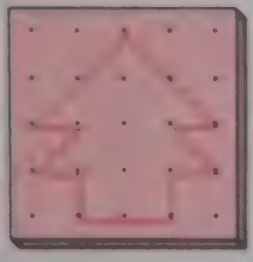
2. 
7 sq units

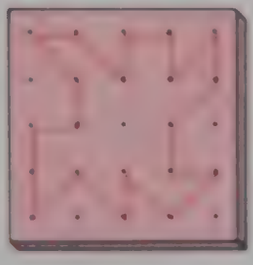
3. 
6 sq units

4. 
$3\frac{1}{2}$ sq units

5. 
4 sq units

6. 
$7\frac{1}{2}$ sq units

7. 
9 sq units

8. 
10 sq units

9. 
7 sq units

Find the area of the shaded region on the geoboards in exercises 10–12.

10. 
12 sq units

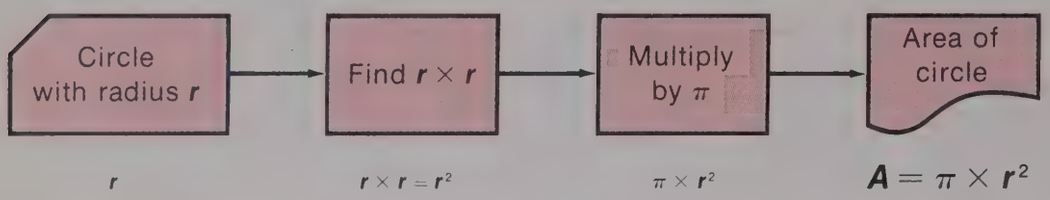
11. 
8 sq units

12. 
9 sq units

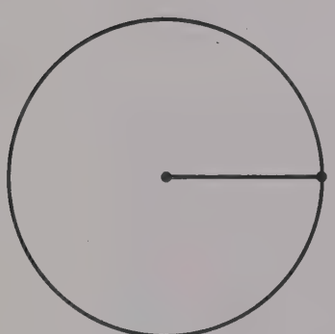
In exercises 10, 11, and 12 children can apply Pick's Theorem twice, subtracting the area of the hole from the larger outer region. The area of the shaded region may also be found by adding the nails on both inner and outer borders, dividing the sum by 2, then adding the number of nails in the shaded part of the region.

Area of Circles

Use a centimeter ruler to measure the radius of each circle. Then find the area of the circle to the nearest tenth centimeter. Use $\pi \approx 3.14$. The flow chart may help you.

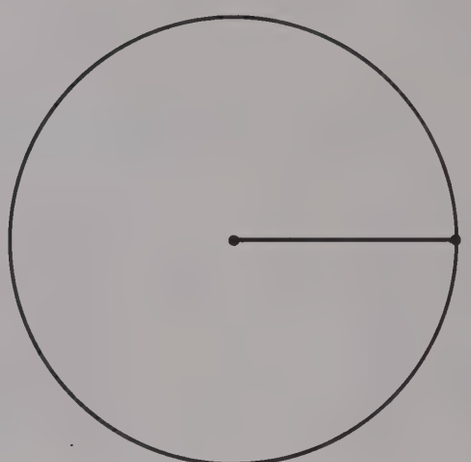


1.



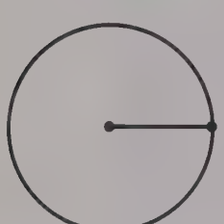
$A =$ 12.6 cm^2

2.



$A =$ 24.6 cm^2

3.



$A =$ 5.3 cm^2

4.

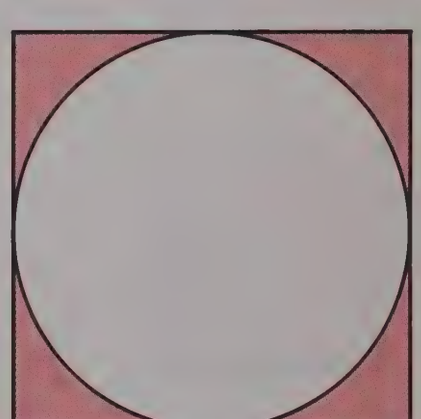
$A =$ 2.0 cm^2

5. A Area of square = 25 cm^2

   B Area of circle = 19.6 cm^2

   C Area of shaded region = 5.4 cm^2

   D Area of circle ÷ area of square: .78

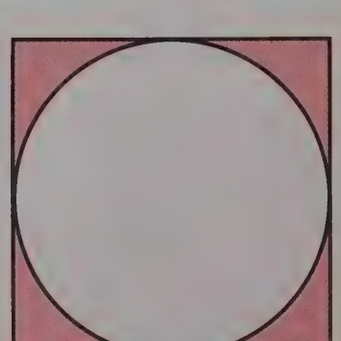


6. A Area of square = 16 cm^2

   B Area of circle = 12.6 cm^2

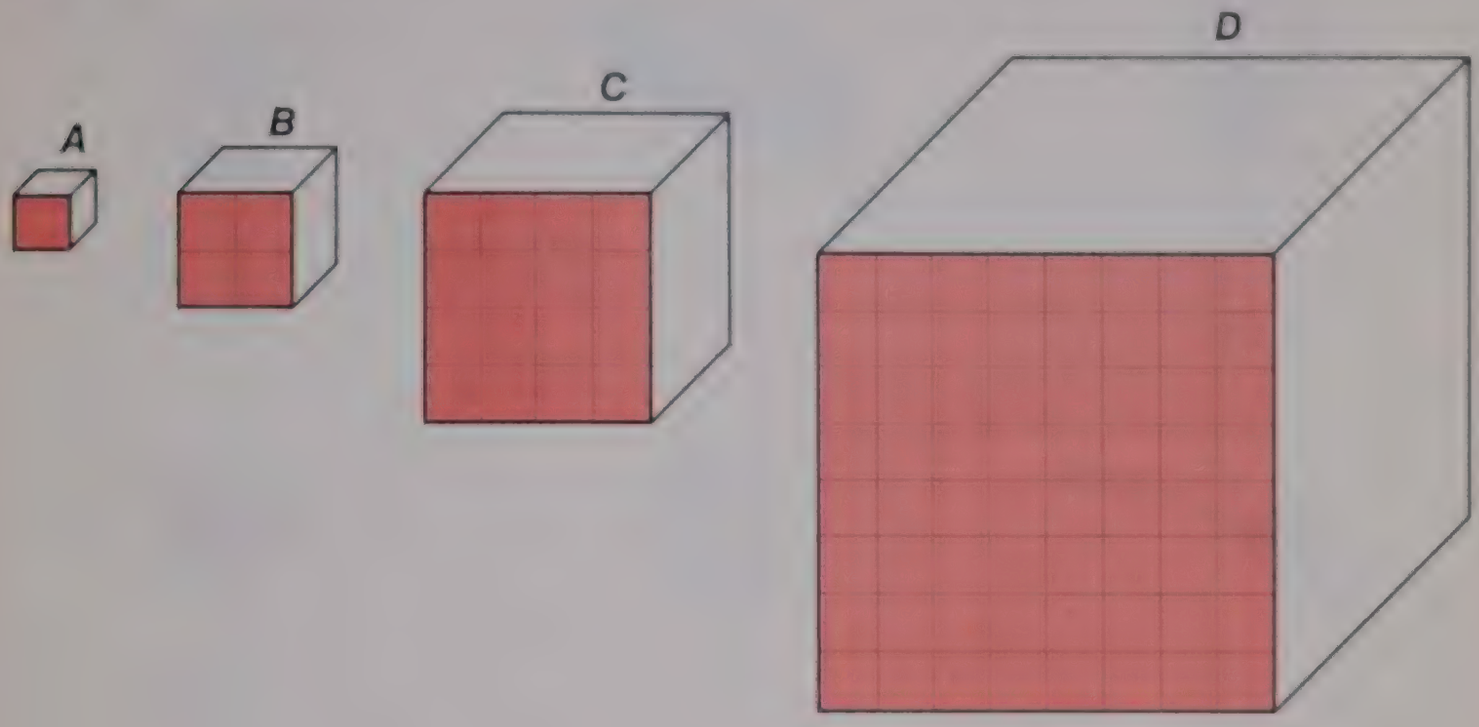
   C Area of shaded region = 3.4 cm^2

   D Area of circle ÷ area of square: .78



Comparing Volumes

These are the boxes a peanut factory uses to ship peanuts to their customers. They are cubic in shape. The colored faces will help you compare sizes—but be careful!



1. Suppose box *B* holds 10 kilograms of peanuts.

   a Which box would you use to ship 100 kilograms of peanuts? D

   b What is the fewest number of boxes you would need to ship 500 kg of peanuts if each box must be completely filled? 8 boxes 6-C 2-B

   c About how much does box *A* hold? $1\frac{1}{4}$ kg

2. a How many boxes the size of *A* can you put in box *B*? 8

   b How many of box *B* can you place in *C*? 8

   c How many of box *C* can you place in *D*? 8

   d How many of box *A* can you place in *D*? 512

3. Suppose box *B* holds 16 liters. Estimate the capacity of the other boxes

   a Box *A* 2 ℓ b Box *C* 128 ℓ c Box *D* 1024 ℓ

These exercises indicate the relationship between doubling the length of the sides of a cube and its respective volume.

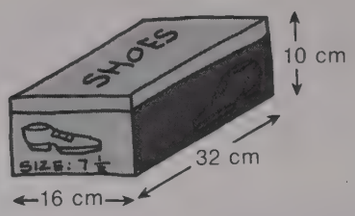
Estimating Capacity

Ring the **best** estimate for the capacity of each object.

1

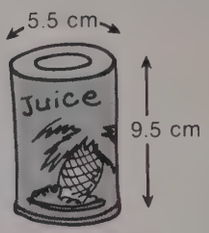
A 1.5 ℓ
B 1.75 ℓ
C 2.0 ℓ

2.



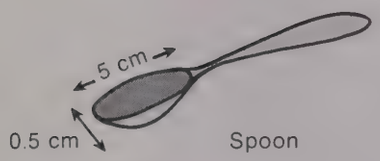
A 3 ℓ
B 4 ℓ
C 5 ℓ

3.



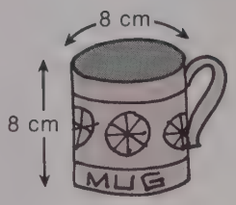
A 150 ml
B 300 ml
C 450 ml

4.



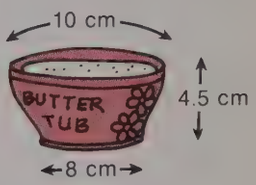
A 5 ml
B 50 ml
C 150 ml

5.



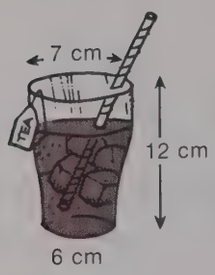
A 200 ml
B 400 ml
C 600 ml

6.



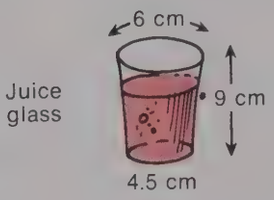
A 100 ml
B 200 ml
C 300 ml

7.



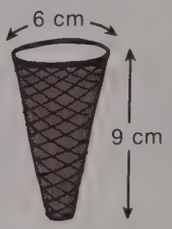
A 0.1 ℓ
B 0.4 ℓ
C 0.7 ℓ

8.



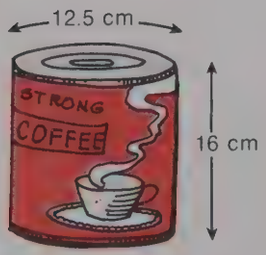
A 40 ml
B 140 ml
C 240 ml

9.



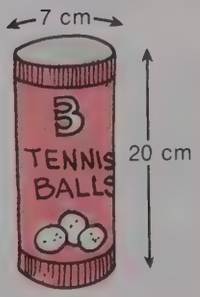
A 80 ml
B 180 ml
C 280 ml

10.



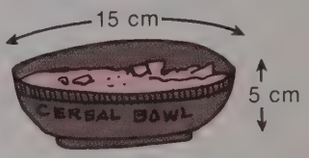
A 1 ℓ
B 2 ℓ
C 3 ℓ

11.



A 500 ml
B 750 ml
C 1 ℓ

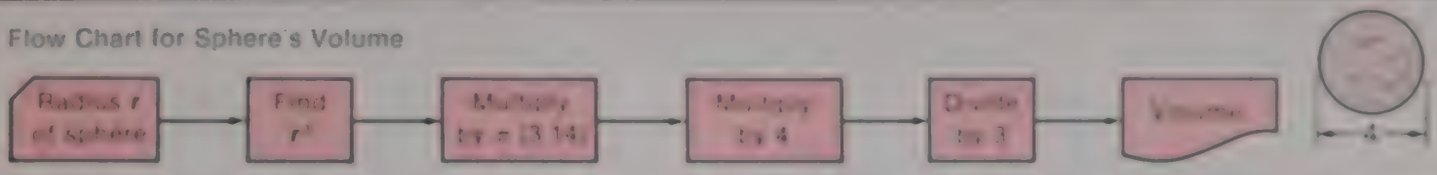
12.



A 100 ml
B 600 ml
C 900 ml

Volume of Spheres

Flow Chart for Sphere's Volume



$r = 2$ | $2^3 = 8$ | $3.14 \times 8 = 25.12$ | $25.12 \times 4 = 100.48$ | $100.48 \div 3 = 33.49$ | 33.49 cubic units

Find the volume of each object in cubic centimeters. Round your answer to the nearest tenth of a cubic centimeter.

1. A small marble:
   $r = 0.5$ cm
   .5 cm^3

2. Golf ball:
   $r = 2.1$ cm
   38.8 cm^3

3. Baseball:
   $r = 3.7$ cm
   212.1 cm^3

4. Softball:
   $r = 4.9$ cm
   492.6 cm^3

5. Volley Ball
   $r = 10.9$ cm
   5421.9 cm^3

6. Basketball
   $r = 12.1$ cm
   7416.9 cm^3

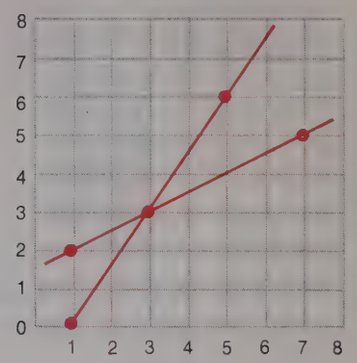
These problems could be solved by children with the aid of desk calculators or minicomputers

Finding the Buried Treasure

Each problem gives the location for a "buried treasure" on the grid. Mark the points, lines, and segments on the grid. Then give the coordinates for the "treasure."

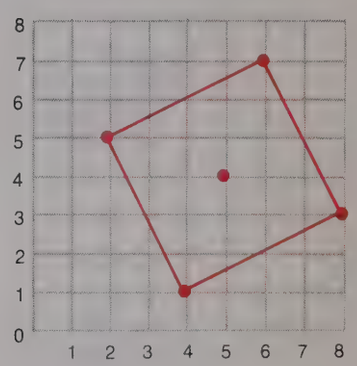
1. The treasure is found at the intersection of the line through (1, 2) and (7, 5) and the line through (5, 6) and (1, 0).

Treasure is at (3, 3)



2. The treasure is at the center of a square whose corners are at (2, 5), (6, 7), (8, 3), and (4, 1).

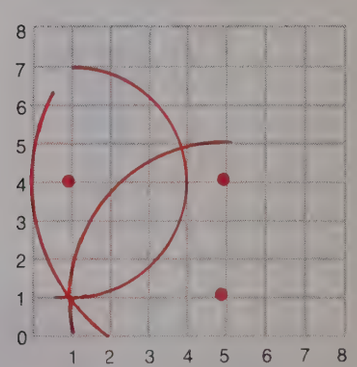
Treasure is at (5, 4)



3. The treasure is 3 units from (1, 4), 4 units from (5, 1), and 5 units from (5, 4).

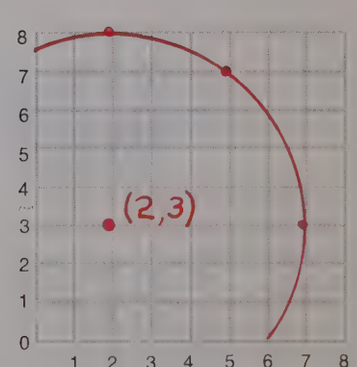
Treasure is at (1, 1)

Compass is needed to find answer.



4. The treasure is at the center of the circle that passes through (2, 8), (5, 7), (7, 3).

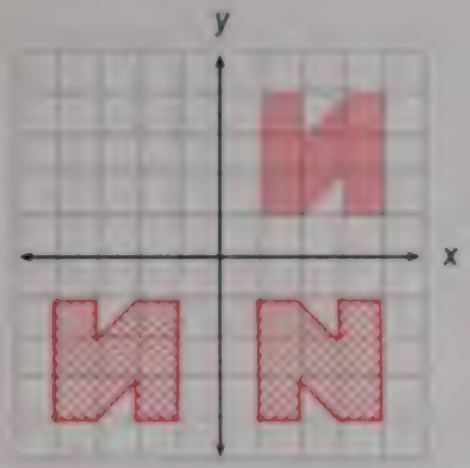
Treasure is at (2, 3)



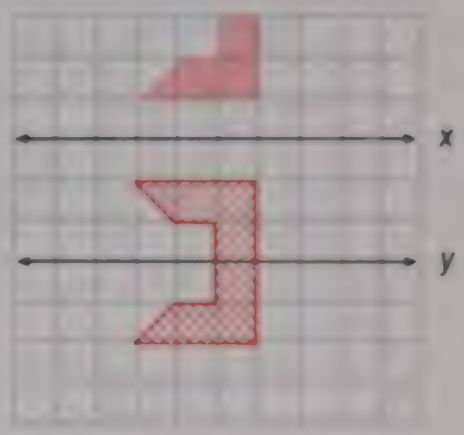
Reflections

Draw the reflection image of the figure in line **x**. Then reflect that image in line **y**.

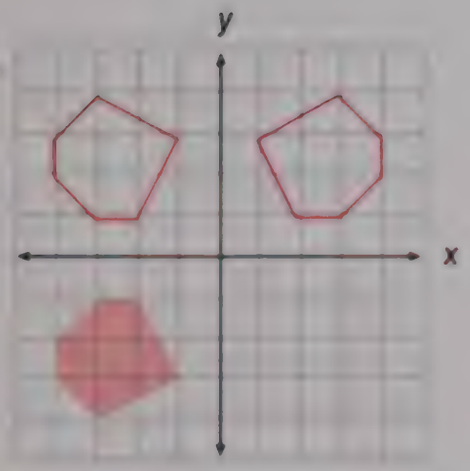
1.



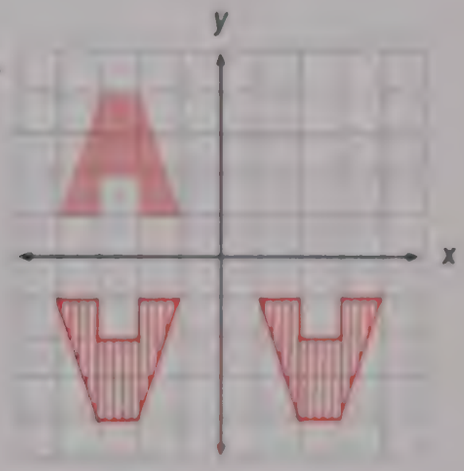
2.



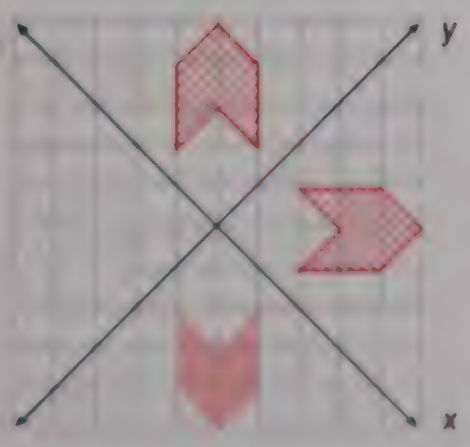
3.



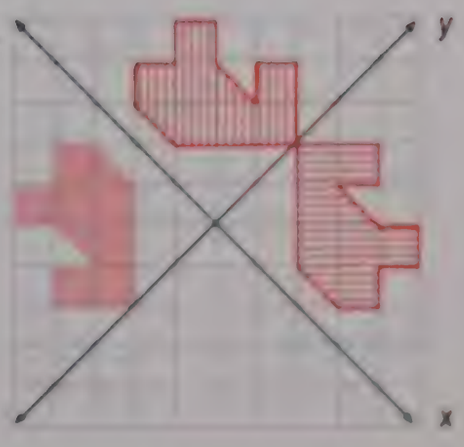
4.



5.



6.

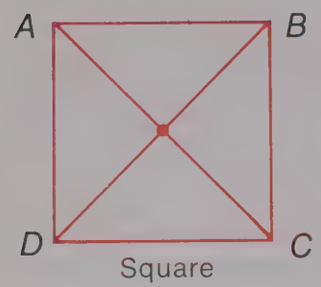


Plastic or nonbreakable mirrors can be used by children to find the reflections and check their work.

Rotations

For each rotation with center at 0, give the image of each point of the figure.

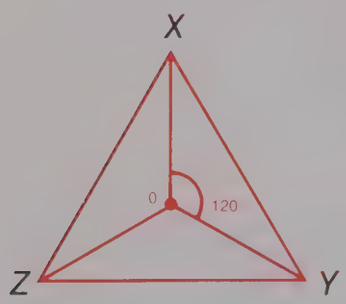
1.



Square

90° clockwise		180° clockwise		270° clockwise	
Point	Image	Point	Image	Point	Image
A	B	A	C	A	D
B	C	B	D	B	A
C	D	C	A	C	B
D	A	D	B	D	C

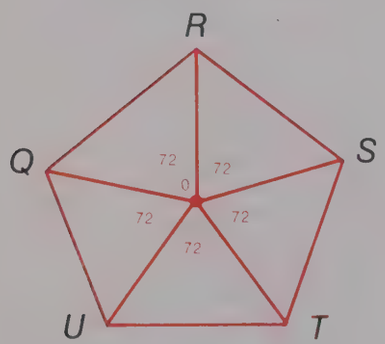
2.



Equilateral Triangle

120° clockwise		120° counterclockwise	
Point	Image	Point	Image
X	Y	X	Z
Y	Z	Y	X
Z	X	Z	Y

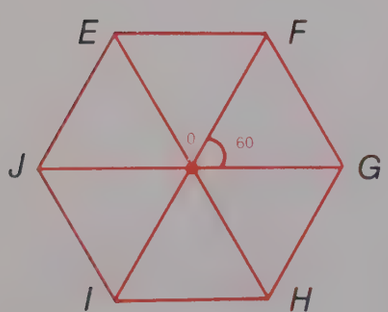
3.



Regular Pentagon

72° counterclockwise		216° clockwise	
Point	Image	Point	Image
Q	U	Q	T
R	Q	R	U
S	R	S	Q
T	S	T	R
U	T	U	S

4. Choose 2 rotations of your own using the regular hexagon and complete the tables.
Answers will vary. Rotations should be multiples of 60.



Regular Hexagon

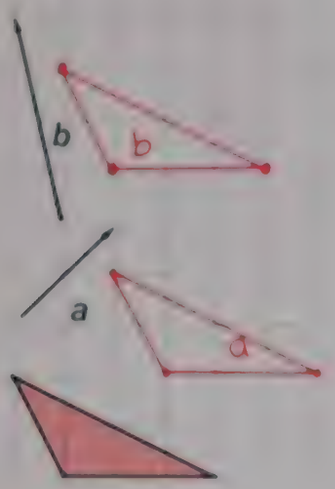
Point	Image	Point	Image
E		E	
F		F	
G		G	
H		H	
I		I	
J		J	

Provide tracing paper for children to use in finding the rotational images.

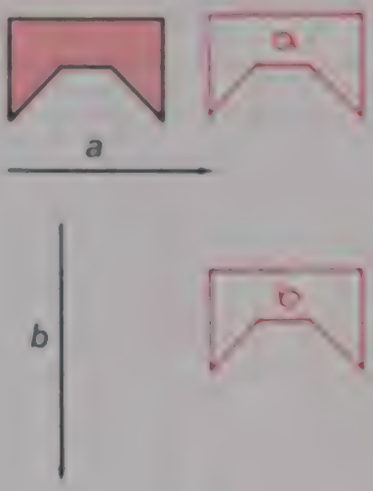
Translations

Show the slide image of each figure for arrow **a**, then show the final figure which results when image is moved the distance and direction of arrow **b**

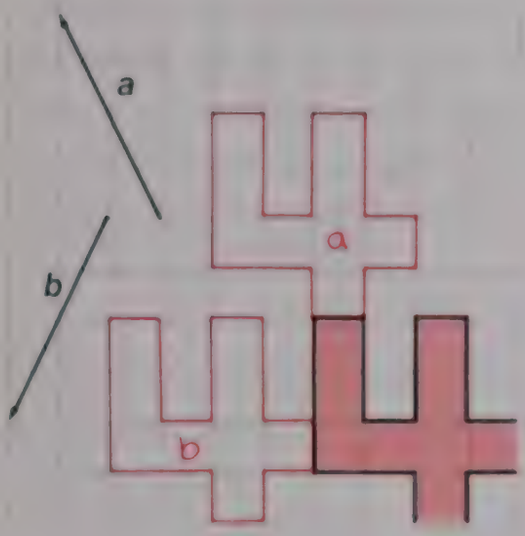
1.



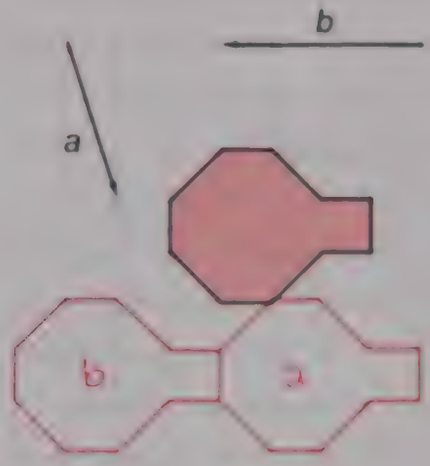
2.



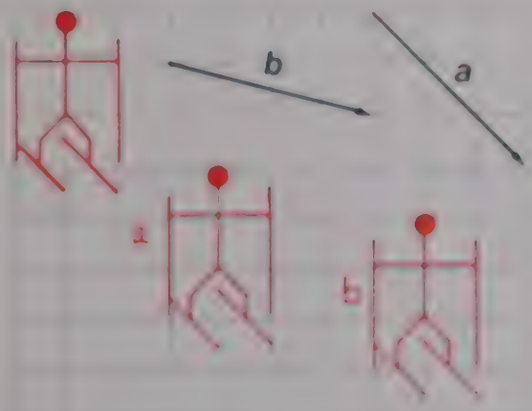
3.



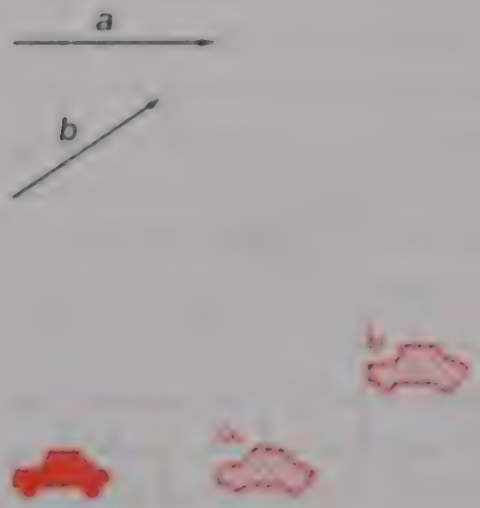
4.



5.



6.

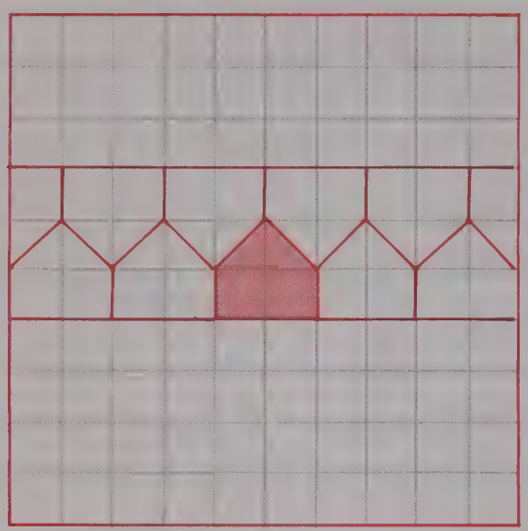


Tesselations

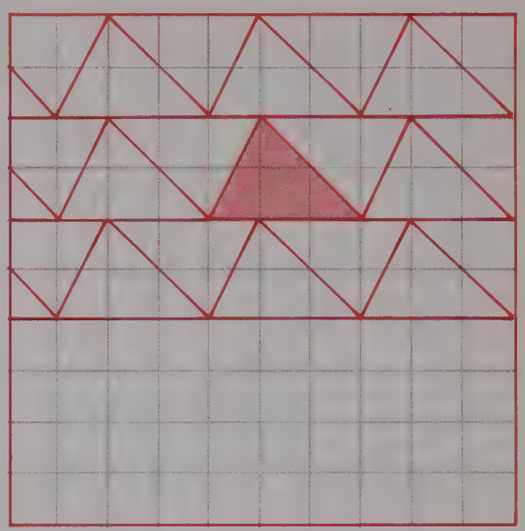
Tesselate each square region with the shape given in the square.

A portion of each tesselation is shown

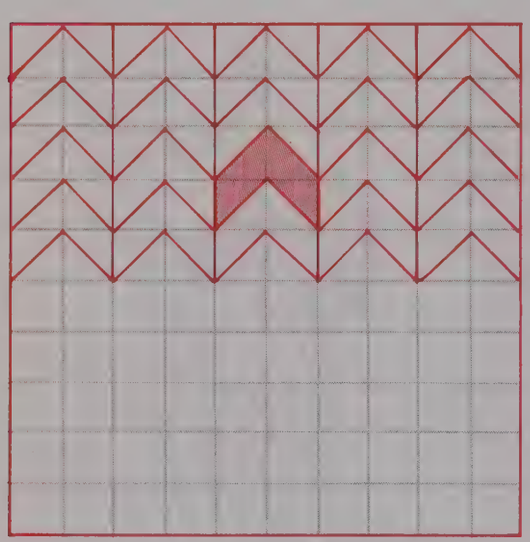
1.



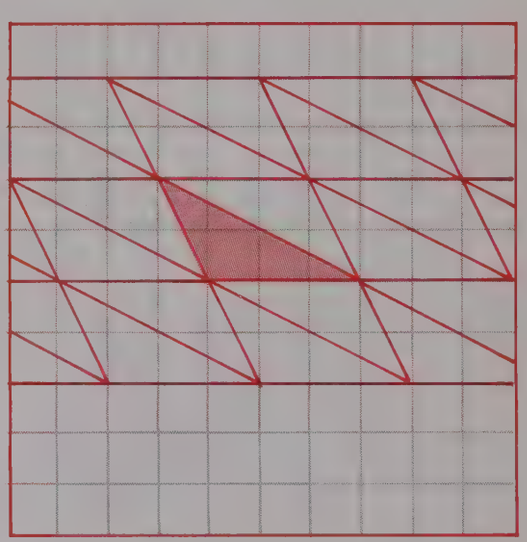
2.



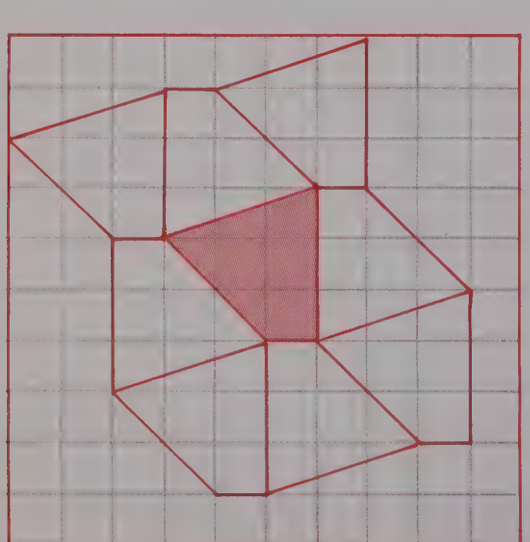
3.



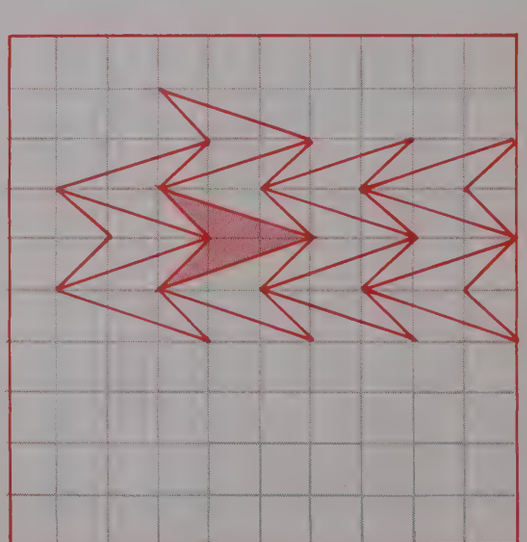
4.



5.

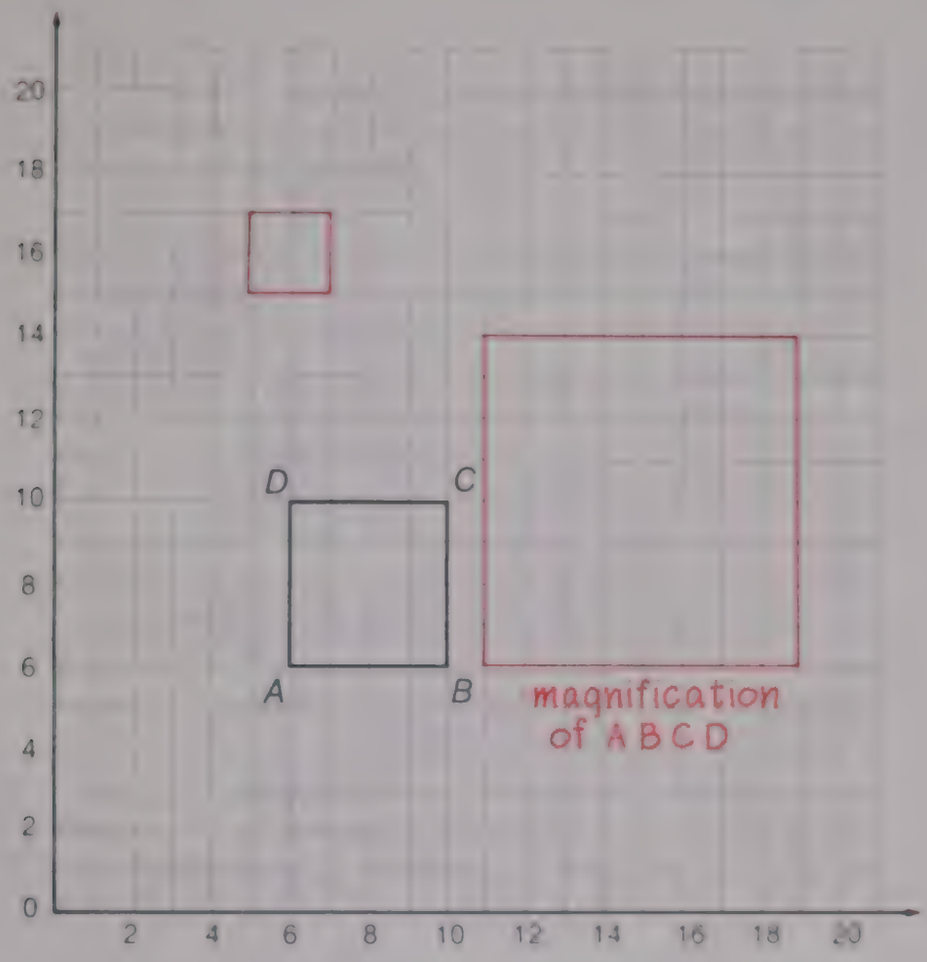


6.



Be sure that children understand that the given regions will tesselate the square regions but not cover them exactly. Portions of the tesselating figure may extend over the boundary of the square.

Magnifying and Shrinking Figures



1. ▲ What is the perimeter of square *ABCD*? 16 units

   ■ What is its area? 16 sq. units

2. Draw a magnification of square *ABCD* using a scale factor of 2.

   ▲ The "new" perimeter is how many times the perimeter of *ABCD*? 2 times

   ■ The new area is how many times the area of *ABCD*? 4 times

3. Draw a smaller square by using a scale factor of $\frac{1}{2}$.

   ▲ How does this new perimeter compare to the perimeter of *ABCD*? $\frac{1}{2}$ of ABCD

   ■ How does this new area compare to the area of *ABCD*? $\frac{1}{4}$ of ABCD

4. Compare the smallest square to the largest square.

   Perimeter 8 and 32 $(\frac{1}{4})$ Area 4 and 64 $(\frac{1}{16})$

Ratio

In Exercises 1–5 find lengths ***a*** and ***b*** of each pair of segments. Then give the ratio of ***a*** to ***b***.

1. a

   b

   a = 2 cm

   b = 5 cm

   a : ***b*** = 2:5

2. a

   b

   a = 3 cm

   b = 10 cm

   a : ***b*** = 3:10

3. a

   b

   a = 3 cm

   b = 2 cm

   a : ***b*** = 3:2

4. a

   b

   a = 5 cm

   b = 6 cm

   a : ***b*** 5:6

5. a

   b

   a = 4 cm

   b = 8 cm

   a : ***b*** = 4:8 or 1:2

In Exercises 6–10 find length ***a***, then draw a segment whose length ***b*** will be in the given ratio to length ***a***.

6. a

   a = 3 cm

   a : ***b*** = 3 to 4

   b = 4 cm

7. a

   a = 5 cm

   a : ***b*** = 1 to 2

   b = 10 cm

8. a

   a = 9 cm

   a : ***b*** = 3 to 1

   b = 3 cm

9. a

   a = 5 cm

   a : ***b*** = 5 to 7

   b = 7 cm

10. a

    a = 2.5 cm

    a : ***b*** = 1 to 4

    b = 10 cm

Centimeter rulers will need to be available for the activity on this page.

Scale Drawings

Measure the map distance between the cities to the nearest tenth cm
Then use the scale to find the actual distance to the nearest ten kilometers.

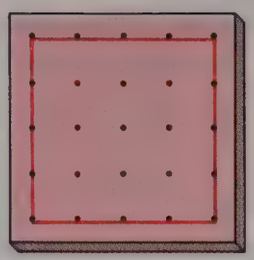
Measurements may vary

2 cm	1. London–Paris	350 km	8	8. Paris–Lisbon	1400 km
2.8	2. Lisbon–Madrid	490 km	8.2	9. Rome–London	1440 km
5.2	3. Berlin–London	910 km	2.6	10. Bern–Paris	460 km
5.9	4. Paris–Copenhagen	1030 km	4.2	11. Berlin–Bern	740 km
7.8	5. Madrid–Rome	1370 km	8.7	12. Rome–Copenhagen	1520 km
2.5	6. Dublin–London	440 km	6.6	13. Brussels–Rome	1160 km
1.6	7. Brussels–Paris	280 km	13.8	14. Lisbon–Copenhagen	2420 km

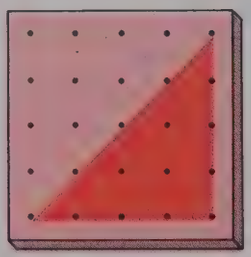
Centimeter rulers will need to be available for this activity on this page.

Percents

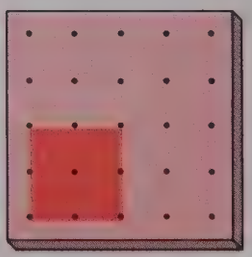
The large square region covers 100% of the geoboard region.



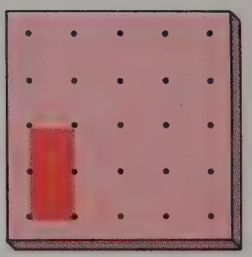
What percent of the geoboard region does each of the shaded regions below cover?

1. 

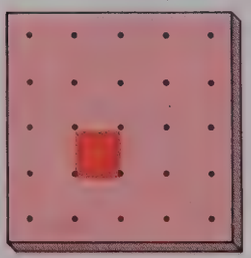
50 %

2. 

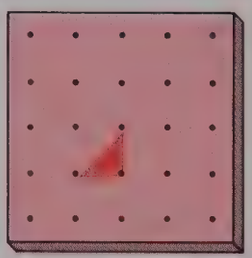
25 %

3. 

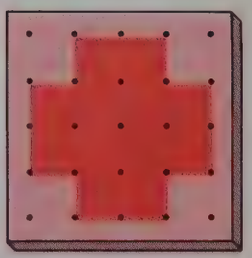
$12\frac{1}{2}$ %

4. 

$6\frac{1}{4}$ %

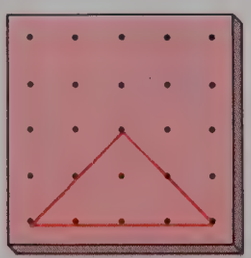
5. 

$3\frac{1}{8}$ %

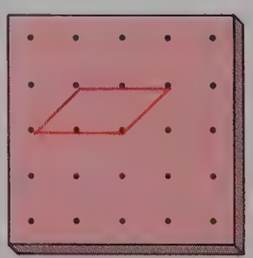
6. 

75 %

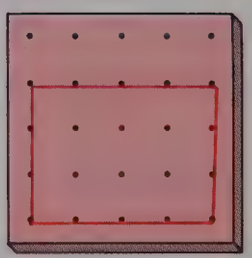
Draw a region covering the given percent of the geoboard.

7. 

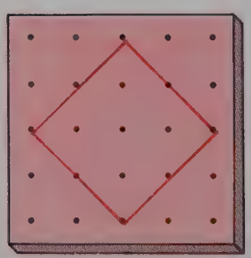
A triangular region covering 25%

8. 

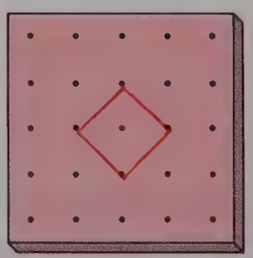
A parallelogram covering $12\frac{1}{2}$%

9. 

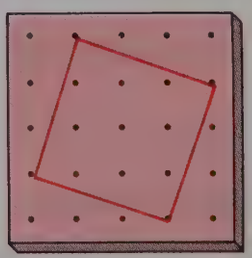
A rectangle covering 75%

10. 

A square covering 50%

11. 

A square covering $12\frac{1}{2}$%

12. 

A square covering $62\frac{1}{2}$%

Students should use actual geoboards to find answers, then record their figures on the geoboards pictured on this page.

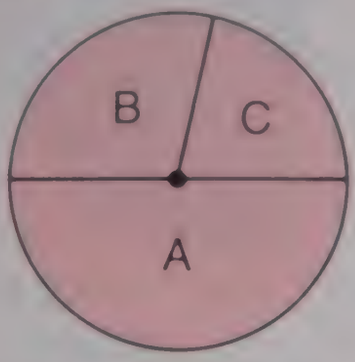
Estimating Percents

1. Write your estimate of the percent of the circle for each part shown.

A 50

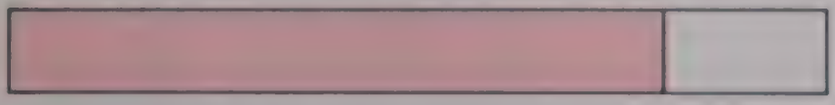
B 30

C 20

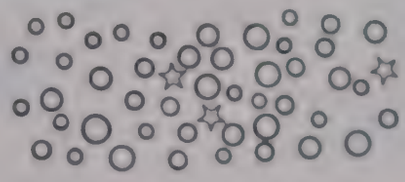


2. A About what percent of the strip is shaded? 80 %

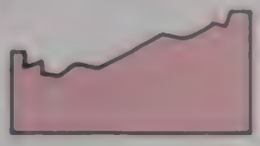
B About what percent is not shaded? 20 %



3. What percent of the objects are stars? 6 %



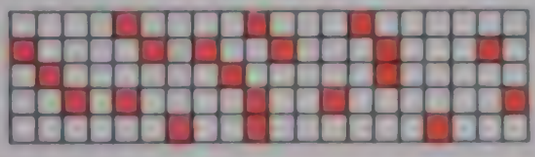
4. What percent of the square region is left? 20-30%



5. What percent of the eggs are broken? 25 %

6. What percent of the bowling pins are standing? 30 %

7. What percent of the tiles are shaded? 20 %



8. What percent of the students in your class wear glasses? ___ % answers will vary

9. What percent of your classmates are left-handed? ___ % answers will vary

Estimates in several exercises will vary

Buying on Sale

Find the sale price of each item.

1. **25% off**

Reg. $8.00

SHOULDER BAG

Sale Price $6.00

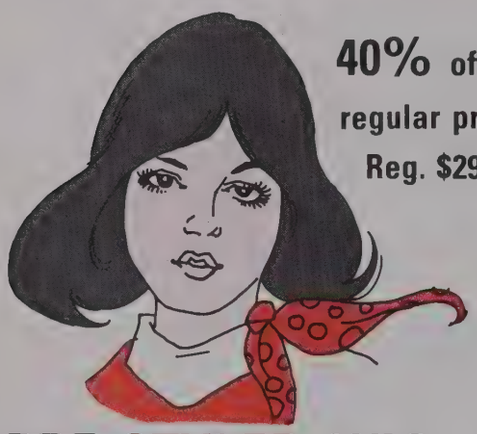
2. **Save 10%**

Reg. $4.96

Sale Price $4.46

GREAT STYLES IN BLUE DENIM JEANS

3. **40% off regular price**

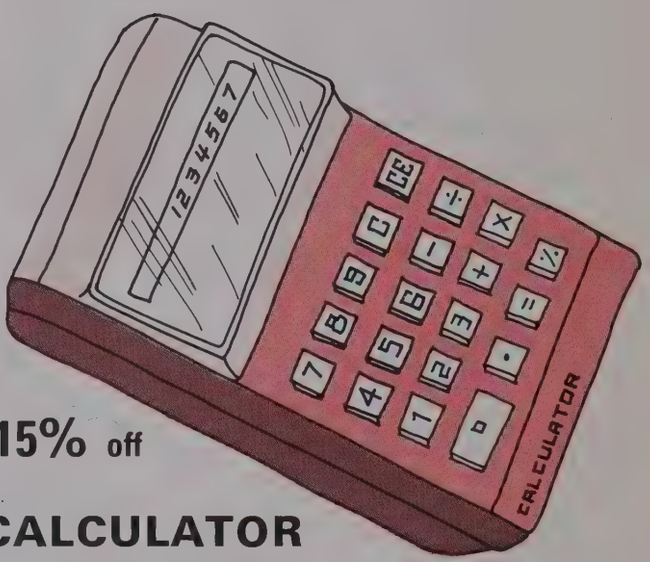


Reg. $29.88

PRE-STYLED WIG

Sale Price $17.93

4. **15% off**



CALCULATOR

Reg. $49.96

Sale Price $42.47

5. **Reduced 20%**

Reg. $35.00

Sale Price $28.00

10-SPEED RACING BIKES

6. **12½% off**

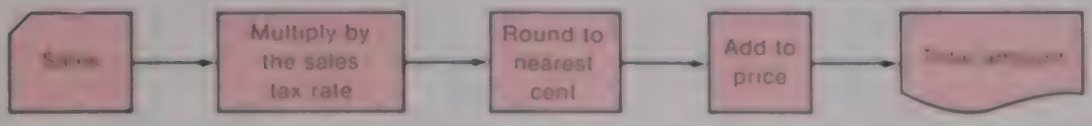
Reg. $80.00

Sale Price $70.00

Sales Tax

Problem: What is the total amount you must pay for the LP record if the sales tax rate is 6% (.06)?
Study the flow chart below to learn how to find the total amount

$4.68 ea.



$4.68 → $4.68 × 06 = $.2808 → $.2808 ≈ $.28 → $4.68 + .28 → $4.96

Find the total amount for each of these items

1. **TENNIS RACKET**
$9.99

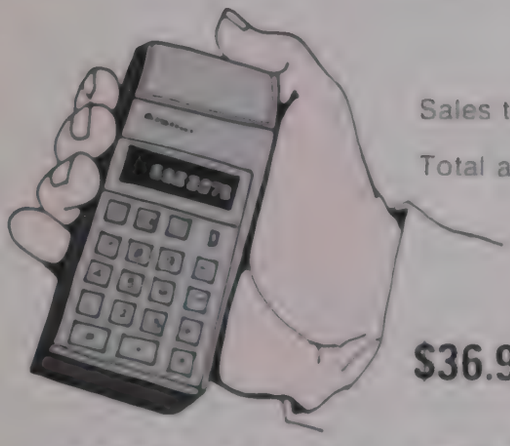
Sales tax rate 5%
Total amount $10.49

2. **DIGITAL CLOCK RADIO**

$24.87

Sales tax rate 4%
Total amount $25.86

3.



Sales tax rate 6%
Total amount $39.16

$36.94

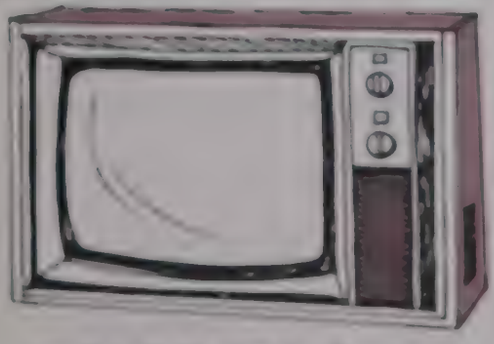
MINI-CALCULATOR

4.

17 jewel mesh bracelet watch —$58.90

Sales tax rate 3%
Total amount $60.67

5.

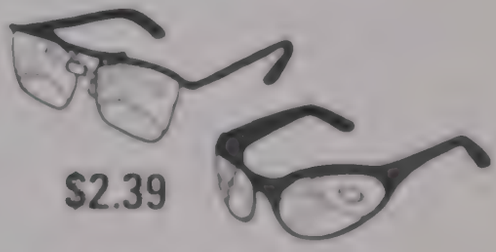


Sales tax rate 5%
Total amount $388.37

$369.88

COLOR TV SET

6.



$2.39

SUNGLASSES

Sales tax rate 6%
Total amount $2.53

A Probability Experiment

Cut a small strip of cardboard just as long as the parallel lines are apart. 
Drop the cardboard strip on the parallel lines and tally whether it touches one of the parallel lines or misses. Keep the tally for 100 tosses.

Touches	
Misses	

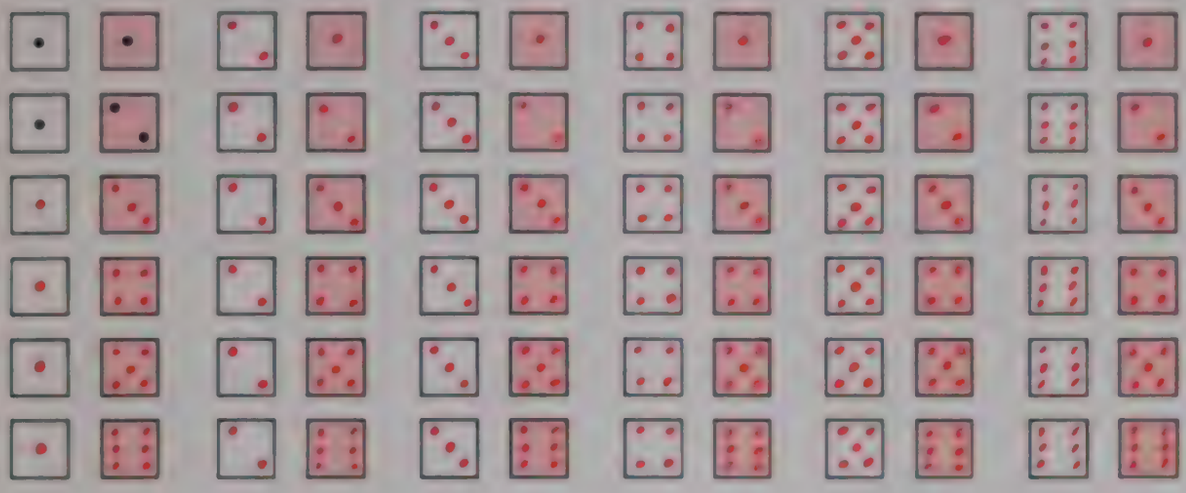
Find this ratio: $\frac{2 \times \text{number of tosses}}{\text{number of touches}} =$ answers will vary but should approximate π

This ratio should be approximately equal to π. Did you come close?
Check your result with your classmates.

You may wish to list on the chalkboard the experimental results found by several children. The combined results should give an improved approximation for π.

Probability with Dice

1. Suppose you have two dice One is white, the other red There are 36 different possible outcomes when the two dice are tossed
How many of these outcomes can you show on the dice faces below?



2. A How many outcomes above have a total of 8 dots on the two faces? 5

   B The chances of getting an 8 are 5 in 36

   C The probability of getting an 8 is $\frac{5}{36}$

3. A How many outcomes above result in a sum of 7? 6

   B The chances of getting 7 are 6 in 36

   C The probability of getting a 7 is $\frac{1}{6}$

   D In 60 tosses of the two dice about how many 7's would you expect to get? 10

4. A How many outcomes have a sum of 12? 1

   B The probability of getting 12 is $\frac{1}{36}$

5. Which sum will occur more often – 5 or 10? 5

6. A How many outcomes result in "doubles" (both dice have the same number of dots)? 6

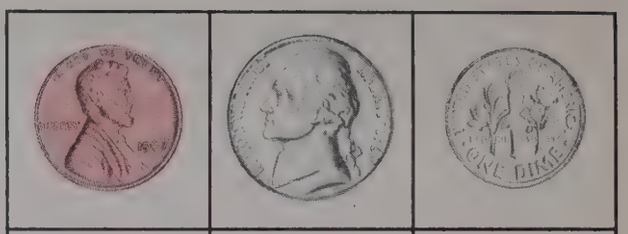
7. Suppose two dice are tossed 1000 times Estimate the number of times the sum (outcome) would occur in the 1000 tosses

A 2 28	D 5 111	G 8 139	J 11 56
B 3 56	E 6 139	H 9 111	K 12 28
C 4 83	F 7 167	I 10 83	

A Coin Toss Experiment

1. Suppose 3 coins, a penny, a nickel, and a dime are tossed at the same time and Heads or Tails are noted on each coin.

- **A** Complete the table to show all the possible outcomes.
- **B** How many outcomes in all? 8
- **C** What is the probability of getting 3 heads? $\frac{1}{8}$
- **D** What is the probability of getting exactly 2 heads? $\frac{3}{8}$
- **E** What is the probability of getting exactly 2 tails? $\frac{3}{8}$
- **F** What is the probability of getting all tails? $\frac{1}{8}$



H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

2. Toss three coins at a time for 100 times. Make a tally of the outcomes at the right.

Outcome	Tally
3 heads	
2 heads	
2 tails	
3 tails	

3. The probability of getting 3 heads is $\frac{1}{8}$. $\frac{1}{8} \times 100$ is about 12. Did you get more or less than 12 outcomes of 3 heads? answers will vary

How many? _____

4. **A** The probability of getting 2 heads is $\frac{3}{8}$. $\frac{3}{8} \times 100$ is about 38.

Did you get more or less than the number in A? answer will vary

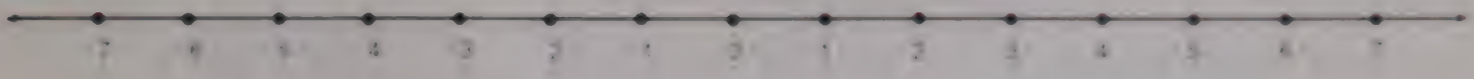
5. **A** The probability of getting 2 tails is $\frac{3}{8}$.

B About how many outcomes should you expect to get 2 tails? 38

C Did you get more or less than the number in B? _____

6. If you tossed 3 coins 800 times, about how many times would you expect to get all coins to come up tails? 100

Patterns in Addition of Integers



Complete each set of equations.

1.

6 + 4 = 10	6 + 1 = 5	6 + 6 = 0
6 + 3 = 9	6 + 2 = 4	6 + 7 = −1
6 + 2 = 8	6 + 3 = 3	6 + 8 = −2
6 + 1 = 7	6 + 4 = 2	6 + 9 = −3
6 + 0 = 6	6 + 5 = 1	6 + 10 = −4

2.

5 + 5 = 0	5 + 0 = −5	5 + 5 = −10
5 + 4 = −1	5 + 1 = −6	5 + 6 = −11
5 + 3 = −2	5 + 2 = −7	5 + 7 = −12
5 + 2 = −3	5 + 3 = −8	5 + 8 = −13
5 + 1 = −4	5 + 4 = −9	5 + 9 = −14

3.

0 + 7 = 7	5 + 2 = 7	10 + −3 = 7
1 + 6 = 7	6 + 1 = 7	11 + −4 = 7
2 + 5 = 7	7 + 0 = 7	12 + −5 = 7
3 + 4 = 7	8 + −1 = 7	13 + −6 = 7
4 + 3 = 7	9 + −2 = 7	14 + −7 = 7

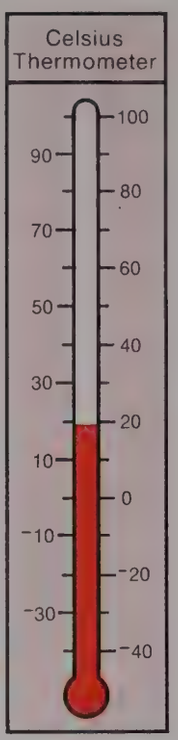
4.

5 + 5 = 0	5 + 4 = 1	5 + 14 = 9
5 + 6 = 1	5 + 3 = 2	5 + 13 = 8
5 + 7 = 2	5 + 2 = 3	5 + 12 = 7
5 + 8 = 3	5 + 1 = 4	5 + 11 = 6
5 + 9 = 4	5 + 0 = 5	5 + 10 = 5

Even though the children may discover a pattern in each problem, they may find it difficult to explain what the pattern is or how they found it. Discovery is more important here than verbalization.

Temperature Problems

1. The temperature at noon was 3°C. By midnight the temperature had fallen 8°. What was the temperature at midnight? $-5°C$



2. One winter day the high temperature for the day was $^{-}10°C$. The low temperature was 12° below the high temperature.

   What was the low temperature? $-22°C$

3. Normal body temperature is 37°C.
   Normal room temperature is 20°C.
   How much higher is body temperature? $17°$

4. Miami, Florida had a temperature of 22°C. On the same day Bemidji, Minnesota had a temperature of $^{-}23°C$. How many degrees

   colder was it in Bemidji? $45°$

5. On a very hot summer day the temperature might be as high as 43°C. On a very cold winter day the temperature might be as low as $^{-}29°C$.

   How many degrees warmer is the summer temperature? $72°$

6. The sunlit side of the planet Mercury has a temperature of 480°C. The dark side of the planet has a temperature of 660° lower than the sunlit

   side. What is the temperature of the dark side of Mercury? $-180°C$

7. The temperature on a cool day was $^{-}2°C$. The temperature fell 9°.

   What was the new temperature? $-11°C$

8. After a very cold low temperature of $^{-}21°$ the temperature rose 12°.

   What was the new temperature? $-9°C$

9. About 95% of the world's population live in parts of the world whose average temperature is between 4.5°C and 27°C. What is the difference

   in these average temperatures? $22.5°C$

10. The boiling point of water is 100°C. How many degrees lower is normal

    body temperature? (See Exercise 3) $63°$

This page applies positive and negative numbers to temperatures.

Multiplication and Division of Integers

Find the missing products in each pattern. Then give three more multiplication equations for each pattern.

Pattern A
$4 \times 4 = 16$
$4 \times 3 = 12$
$4 \times 2 = 8$
$4 \times 1 = 4$
$4 \times 0 = 0$
$4 \times {}^{-}1 = {}^{-}4$
$4 \times {}^{-}2 = -8$
$4 \times {}^{-}3 = -12$
$4 \times -4 = -16$
$4 \times -5 = -20$
$4 \times -6 = -24$

Pattern B
${}^{-}1 \times 6 = {}^{-}6$
${}^{-}1 \times 5 = {}^{-}5$
${}^{-}1 \times 4 = {}^{-}4$
${}^{-}1 \times 3 = -3$
${}^{-}1 \times 2 = -2$
${}^{-}1 \times 0 = 0$
${}^{-}1 \times {}^{-}1 = 1$
${}^{-}1 \times {}^{-}2 = 2$
$-1 \times -3 = 3$
$-1 \times -4 = 4$
$-1 \times -5 = 5$

Pattern C
${}^{-}3 \times {}^{-}6 = 18$
${}^{-}3 \times {}^{-}5 = 15$
${}^{-}3 \times {}^{-}4 = 12$
${}^{-}3 \times {}^{-}3 = 9$
${}^{-}3 \times {}^{-}2 = 6$
${}^{-}3 \times {}^{-}1 = 1$
${}^{-}3 \times 0 = 0$
${}^{-}3 \times 1 = -3$
$-3 \times 2 = -6$
$-3 \times 3 = -9$
$-3 \times 4 = -12$

1. Complete each equation.

   A Since ${}^{-}4 + {}^{-}4 =$ -8, then $2 \times {}^{-}4 =$ -8.

   B Since ${}^{-}5 + {}^{-}5 + {}^{-}5 =$ -15, then $3 \times {}^{-}5 =$ -15.

   C Since ${}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 =$ -8, then $4 \times {}^{-}2 =$ -8.

   D Since $6 \times 6 \times 6 =$ 18, then $3 \times 6 =$ 18.

   E Since ${}^{-}8 + {}^{-}8 + {}^{-}8 + {}^{-}8 =$ -32, then $4 \times {}^{-}8 =$ -32.

2. Complete each division equation.

   A Since $2 \times {}^{-}4 = {}^{-}8$, we know that ${}^{-}8 \div 2 =$ -4 and ${}^{-}8 \div {}^{-}4 =$ -2

   B Since $3 \times {}^{-}5 = {}^{-}15$, we know that ${}^{-}15 \div 3 =$ -5 and ${}^{-}15 \div {}^{-}5 =$ 3

   C Since $4 \times {}^{-}2 = {}^{-}8$, we know that ${}^{-}8 \div 4 =$ -2 and ${}^{-}8 \div {}^{-}2 =$ 4

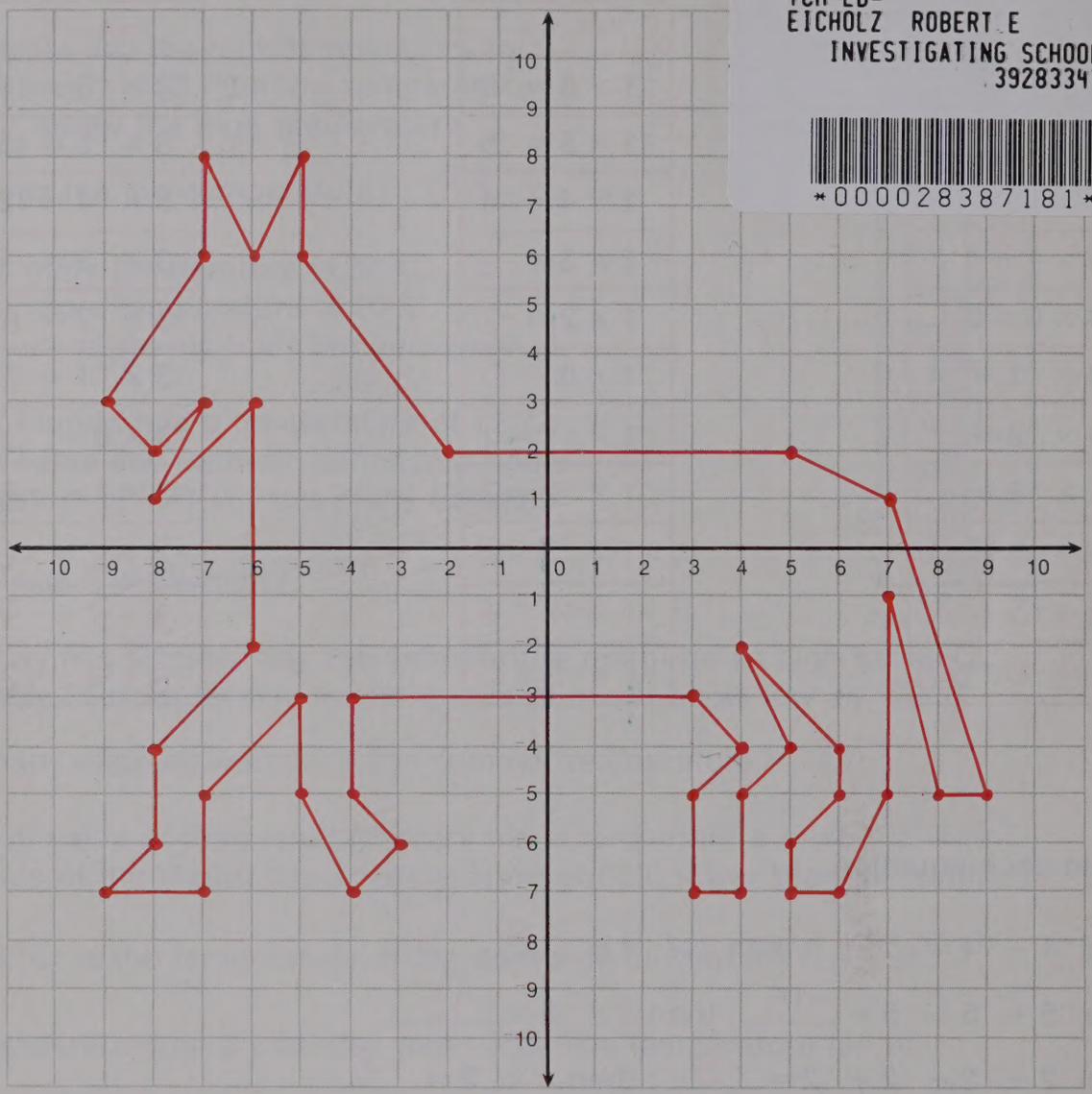
   D Since $3 \times 6 = 18$, we know that $18 \div 3 =$ 6 and $18 \div 6 =$ 3

   E Since $4 \times {}^{-}8 = {}^{-}32$, we know that ${}^{-}32 \div 4 =$ -8 and ${}^{-}32 \div {}^{-}8 =$ 4

Children may discover that the product of a positive integer and a negative integer is a negative integer. Also the product of two negative integers is a positive integer. Once this pattern is seen, division of integers can be readily understood.

Graphing Ordered Pairs of Integers

Connect the points for each ordered pair of integers with segments in the order listed to form a picture.



(7, 1), (8, $^{-}$2), (9, $^{-}$5), (8, $^{-}$5), (7, $^{-}$1), (7, $^{-}$5), (6, $^{-}$7), (5, $^{-}$7), (5, $^{-}$6), (6, $^{-}$5), (6, $^{-}$4), (4, $^{-}$2), (5, $^{-}$4), (4, $^{-}$5), (4, $^{-}$7), (3, $^{-}$7), (3, $^{-}$5), (4, $^{-}$4), (3, $^{-}$3), ($^{-}$4, $^{-}$3), ($^{-}$4, $^{-}$5), ($^{-}$3, $^{-}$6), ($^{-}$4, $^{-}$7), ($^{-}$5, $^{-}$5), ($^{-}$5, $^{-}$3), ($^{-}$7, $^{-}$5), ($^{-}$7, $^{-}$7), ($^{-}$9, $^{-}$7), ($^{-}$8, $^{-}$6), ($^{-}$8, $^{-}$4), ($^{-}$6, $^{-}$2), ($^{-}$6, 3), ($^{-}$8, 1), ($^{-}$7, 3), ($^{-}$8, 2), ($^{-}$9, 3), ($^{-}$7, 6), ($^{-}$7, 8), ($^{-}$6, 6), ($^{-}$5, 8), ($^{-}$5, 6), ($^{-}$2, 2), (5, 2), (7, 1)

Make another graph picture similar to the one above. Make a list of ordered pairs of integers for your picture. See if a classmate can draw your picture using your ordered pairs.

CONTENTS

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